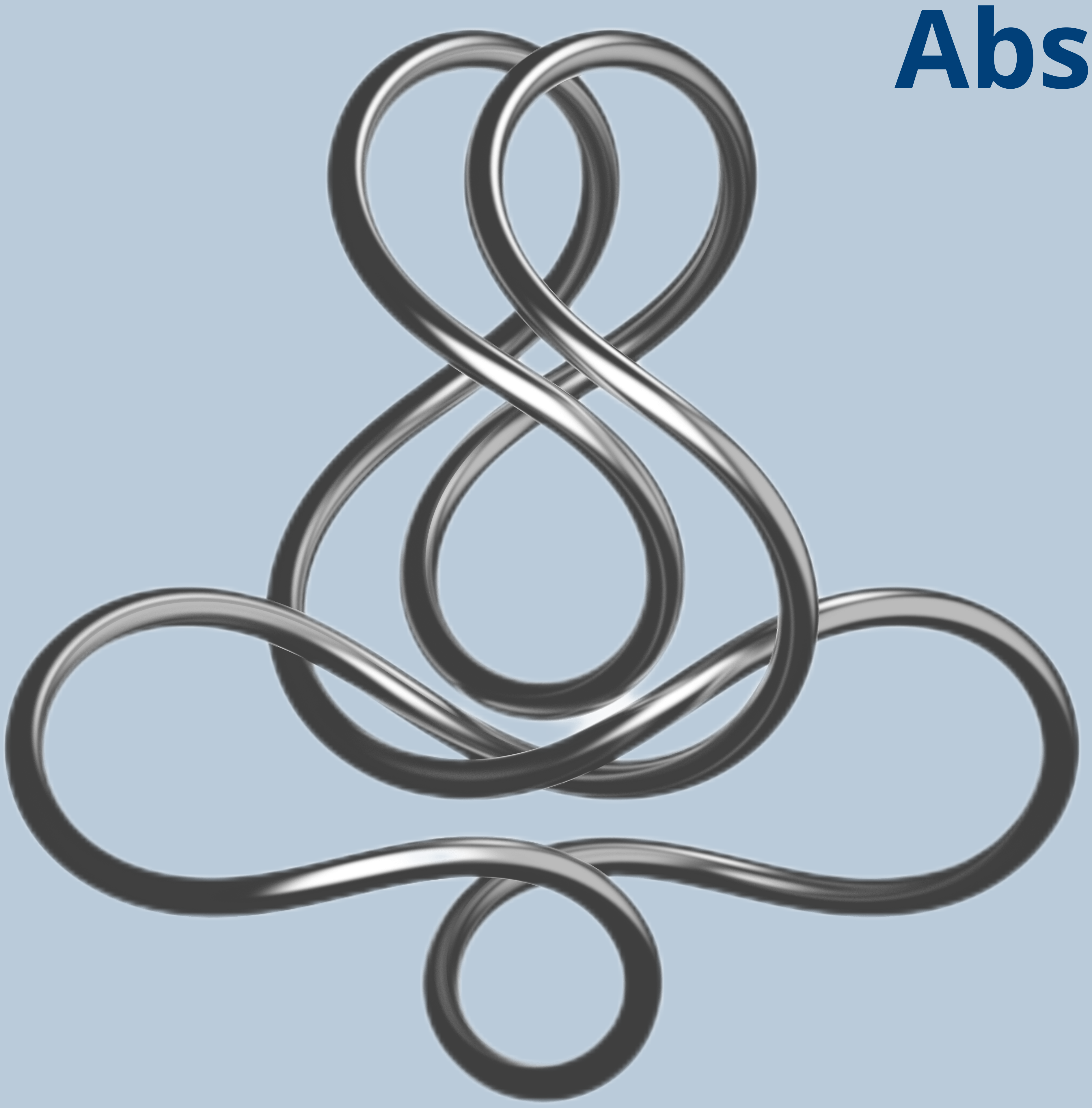

# PADGE 2023

## Pure and Applied Differential Geometry

Dedicated to the memory of Franki Dillen (1963-2013)

Leuven, July 10th – 14th, 2023

# Abstracts





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# Plenary talks

## On constant mean curvature surfaces in product manifolds

Benoît Daniel (joint work with I. Domingos and F. Vitório)
Université de Lorraine

In this talk we will classify constant mean curvature surfaces with constant intrinsic curvature in the product manifolds $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$, where $\mathbb{S}^2$ is the constant curvature sphere and $\mathbb{H}^2$ the hyperbolic plane. We will also consider constant mean curvature surfaces in other homogeneous Riemannian $3$-manifolds, and parallel mean curvature surfaces with constant intrinsic curvature in the product manifolds $\mathbb{S}^2 \times \mathbb{S}^2$ and $\mathbb{H}^2 \times \mathbb{H}^2$. In particular these classifications will provide new examples.

## Positive intermediate Ricci curvature on homogeneous spaces

Miguel Domínguez Vázquez
CITMAga – Universidade de Santiago de Compostela

Positive $k^{\text{th}}$-intermediate Ricci curvature ($\mathrm{Ric}_k > 0$) on a Riemannian manifold of dimension $n$ is a condition that interpolates between positive sectional curvature (when $k = 1$) and positive Ricci curvature (when $k = n-1$). In this talk I will present the main ideas and results of an ongoing project aiming at constructing new examples and deriving certain classifications of compact homogeneous spaces with $\mathrm{Ric}_k > 0$, for $k$ low. The talk is based on joint works with David González-Álvaro, Lawrence Mouillé, Alberto Rodríguez-Vázquez and Jason DeVito.

## (Non-)Compactness for sign-changing solutions of the Yamabe equation at the lowest energy level

Bruno Premoselli (joint work with J. Vétois)
Université Libre de Bruxelles

The Yamabe problem, at the crossroads of differential and conformal geometry, has attracted a lot of attention in the last decades. Recently *sign-changing* solutions of the Yamabe equation have started to be investigated. Despite their seemingly odd nature – sign-changing solutions no longer define a well-defined conformal metric –, sign-changing solutions of the Yamabe equation naturally appear as extremals (when they exist) for the problem of minimizing higher- order eigenvalues of the conformal Laplacian in a given conformal class. The goal of this talk is to describe the behavior of least energy sign-changing solutions of the Yamabe equation. We will review the geometrical origin of such solutions, the existence results known so far and will describe their behavior at the lowest energy level. Our main focus will be a compactness result in small dimensions or in the locally conformally at case recently obtained with J. Vétois (Mc Gill) and its possible application to the extremisation of conformal eigenvalues.

## Infinitely many $p$-harmonic self-maps of spheres

Anna Siffert (joint work with V. Branding)

Westfälische Wilhelms-Universität Münster

We study rotationally $p$-harmonic self-maps between spheres. We prove that for $p \in \mathbb{N}$ given, there exist infinitely many $p$-harmonic self-maps of $\mathbb{S}^m$ for each $m \in \mathbb{N}$ with $p < m < 2 + p + 2\sqrt{p}$.

## Geometric inequalities: methods and applications

Bogdan Suceava

California State University, Fullerton

In 1980, Yu. D. Burago and V.A. Zalgaller published the volume 'Geometric Inequalities', in which it is stated that such relations have many applications "within geometry itself as well as beyond its limits". Since then, there have been several important developments in the area of geometric inequalities, from the investigation of metric spaces to the geometry of Riemannian manifolds. E.g., pursuing a question in the geometry of submanifolds, after the early 1990s, B.-Y. Chen's fundamental inequalities have been investigated by many authors from various viewpoints; we plan to take a closer look at the fundamental methods used in these inquiries, in several geometric contexts.

## On the self-similar solution to the curvature flow for curves

Keti Tenenblat (joint work with H. dos Reis and F. N. da Silva)

Universidade de Brasília

Self-similar solutions to the curvature flow for curves will be considered on the sphere, on the 2-dimensional hyperbolic space and on the 2-dimensional light cone. On the light cone the curvature flow will also be related to the inverse curvature flow, In each case, the geometry of the curves will be described. The talk is based on papers published with my collaborators.

## On the Moebius deformable hypersurfaces

Ruy Tojeiro (joint work with M. I. Jimenez)

ICMC – Universidade de São Paulo

In the article [*Deformations of hypersurfaces preserving the Möbius metric and a reduction theorem*, Adv. Math. **256** (2014), 156–205], Li, Ma and Wang investigated the interesting class of Moebius deformable hypersurfaces, that is, the umbilic-free Euclidean hypersurfaces $f\colon M^n \to \mathbb{R}^{n+1}$ that admit non-trivial deformations preserving the Moebius metric. The classification of Moebius deformable hypersurfaces of dimension $n \geq 4$ stated in the aforementioned article, however, misses a large class of examples. In this talk we report on a recent joint work with M. I. Jimenez, in which we complete that classification for $n \geq 5$.

## Index and stability of minimal submanifolds in the Berger spheres

Francisco Torralbo (joint work with F. Urbano)

Universidad de Granada

A submanifold of a Riemannian manifold is called minimal if its mean curvature vector vanishes everywhere. These manifolds appear as critical points of the volume functional. From this point of view, a compact submanifold is called stable if the second variation of the volume is positive semi-definite. J. Simons proved that the sphere does not admit compact stable minimal submanifolds and he characterized those compact minimal submanifolds which fail short from being stable (the lowest index examples). We will discuss some generalizations of these results to the Berger spheres, that is, odd dimensional spheres endowed with a 1-parameter deformation of the round metric in the direction of the Hopf fibers. We will show that they admit compact stable minimal submanifolds under some conditions over the parameter $\tau$ of the deformation. We will classify the compact stable ones under embeddedness for a special value of the parameter $\tau$.

## Geometry & AI

Alice Barbara Tumpach

Institut CNRS Pauli, Vienna and Lille University

In this talk, we will present some applications of finite-dimensional and infinite-dimensional geometry in the rapidly expanding field commonly known as Artificial Intelligence (AI). The growing need to process geometric data such as curves, surfaces or fibered structures in a resolution-independent way that is invariant to shape-preserving transformations gives rise to mathematical questions both in pure and applied differential geometry, but also in numerical analysis. To illustrate this, applications of shape analysis in medical imaging will be given. On the other hand, the extraction of geometric information from point clouds formed by data sets is an area where differential geometry meets probability, as in dimension reduction or manifold learning. Some of the challenges in these areas will be mentioned.

# Short talks

## Heat flow of $p$-harmonic maps from complete manifolds into regular balls

Zeina Al Dawoud (joint work with A. Fardoun and R. Regbaoui)

University of Brest

We study the heat flow of $p$-harmonic maps between complete Riemannian manifolds. We prove the global existence of the flow when the initial datum has its values in a generalised regular ball. In particular, if the target manifold has nonpositive sectional curvature, we obtain the global existence of the flow for any initial datum with finite $p$-energy. If in addition the target manifold is compact, the flow converges at infinity to a $p$-harmonic map. This gives an extension of the well known results of Li-Tam and Liao-Tam concerning the harmonic heat flow ($p = 2$) to the case $p \geq 2$. We also derive a Liouville type theorem for $p$-harmonic maps between complete Riemannian manifolds when the Ricci curvature of the domain manifold is nonnegative.

## Totally geodesic Lagrangian submanifolds of the pseudo-nearly Kähler $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$

Mateo Anarella (joint work with J. Van der Veken)

KU Leuven

In this talk we will analyze the nearly Kähler structure of the pseudo-Riemannian manifold $\tilde{M} = \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. We can define a natural almost product structure $P$, compatible with the nearly Kähler metric, by swapping the vector fields tangent to each component of $\tilde{M}$. Given a Lagrangian submanifold $M$, we will study the different forms the restriction $P|_{TM}$ can take. We classify, up to isometries, all totally geodesic Lagrangian submanifolds of $\tilde{M}$.

## Generic submanifolds of almost contact metric manifolds

Cornelia-Livia Bejan

"Gh. Asachi" Technical University of Iasi

Ronsse introduced the notion of generic and skew CR-submanifolds of almost Hermitian manifolds in order to unify and generalize the notions of holomorphic, totally real, CR, slant, semi-slant and pseudo-slant submanifolds. Other authors, such as Tripathi, extended this notion to contact geometry, under the name of almost semi-invariant submanifolds. This class includes the one with the same name introduced by Bejancu (and studied also by Tripathi), but without being equal. The class of submanifolds that we introduce and study here in contact geometry is called by us generic submanifolds, in order to avoid the above confusion, and also since it is different from the class studied by Tripathi, because in our paper, the Reeb vector field is not necessarily tangent to the submanifold. We obtain necessary and sufficient conditions for the integrability and parallelism of some eigen-distributions of a canonical structure on generic submanifolds. Some properties of the Reeb vector field to be Killing and its curves to be geodesics are investigated. Totally geodesic and mixed geodesic results on generic submanifolds are established. We give necessary and sufficient conditions for a generic submanifold to be written locally as a product of the leaves of some eigen-distributions. Some examples on both generic submanifolds and skew CR-submanifolds of almost contact metric manifolds are constructed.

## Ricci solitons of submanifolds of product spaces

Burcu Bektaş Demirci

Fatih Sultan Mehmet Vakıf University

A smooth vector field $\xi$ on a Riemannian manifold $(M, g)$ is said to define a Ricci soliton if it satisfies the following Ricci soliton equation:

$$\frac{1}{2}\mathcal{L}_\xi g + \text{Ric} = \lambda g,$$

where $\mathcal{L}_\xi g$ is the Lie derivative of the metric tensor $g$ with respect to $\xi$, Ric is the Ricci tensor of $(M, g)$ and $\lambda$ is a constant, [1]. Here, we call the vector field $\xi$ as potential field.

In this work, we examine Ricci soliton on hypersurface of $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$ whose potential field is the projection of $\frac{\partial}{\partial x_{n+2}}$ on the tangent space of the hypersurface where $(x_1, x_2, \ldots, x_{n+2}) \in \mathbb{E}^{n+2}$. Also, we classify Ricci soliton on class $\mathcal{A}$ hypersurface of such product spaces.

[1] B.-Y. Chen, *A survey on Ricci solitons on Riemannian submanifolds*, Contemporary Mathematics **674** (2016) 27–39.

## PMC biconservative surfaces in complex space forms

Hiba Bibi (joint work with B.-Y. Chen, D. Fetcu and C. Oniciuc)

Université de Tours

We consider PMC surfaces in complex space forms, and study the interaction between the notions of PMC, totally real and biconservative. We first consider PMC surfaces in a non-flat complex space form and prove that they are biconservative if and only if totally real. Then, we find a Simons type formula for a well-chosen vector field constructed from the mean curvature vector field and use it to prove a rigidity result for CMC biconservative surfaces in 2-dimensional complex space forms. We prove then a reduction codimension result for PMC biconservative surfaces in non-flat complex space forms. We conclude by constructing examples of CMC non-PMC biconservative submanifolds from the Segre embedding, and discuss when they are proper-biharmonic.

## Higher parallel transport and the Mackenzie class

Olivier Brahic (joint work with P. Frejlich)

Universidade Federal do Paraná

Looking at a weakened version of a transitive Lie algebroid over a manifold M, we obtain a class in the 3rd De Rham cohomology $[\phi] \in H^3(M, \mathfrak{z}_M)$ of $M$, with values in a flat bundle of abelian Lie algebras. Algebraically, the construction is similar to the canonical class in $H^3(\mathrm{out}(\mathfrak{g}), \mathfrak{z})$ associated with the crossed module of Lie algebras $\mathfrak{g} \to \mathrm{der}(\mathfrak{g})$.

We shall explain how to interpret the various geometric objects involved. In particular one can construct a 3-functor of holonomy, that allows one to see the integrals of $\phi$ as measuring the higher commutativity of cubical diagrams.

## Ricci solitons as submanifolds of complex hyperbolic spaces

Ángel Cidre-Díaz

CITMAga – Universidade de Santiago de Compostela

As a consequence of the recently solved Alekseevsky conjecture [1], and a result lying at the intersection of Ado's theorem for Lie algebras and the Nash embedding theorem [2], any expanding homogeneous Ricci soliton can be found, up to isometry, as a Lie subgroup of the solvable Iwasawa group associated with a symmetric space of non-compact type, considered with the induced metric. Motivated by this fact, we have addressed the classification of homogeneous Ricci solitons arising as Lie subgroups of the solvable Iwasawa groups of complex hyperbolic spaces. We also analyse the minimality of the examples obtained.

[1] C. Böhm, R. Lafuente. Non-compact Einstein manifolds with symmetry. *J. Amer. Math. Soc.* 36 (3), 591-651.
[2] M. Jablonski. Einstein solvmanifolds as submanifolds of symmetric spaces. *arXiv:1810. 11077.*

## Sphere-like isoparametric hypersurfaces in Damek-Ricci spaces

Balázs Csikós (joint work with M. Horváth)

Eötvös Loránd University, Budapest

A hypersurface in a Riemannian manifold is called isoparametric if its nearby parallel hypersurfaces are of constant mean curvature. Harmonic spaces are Riemannian manifolds in which small geodesic spheres are isoparametric hypersurfaces. Damek–Ricci spaces are non-compact harmonic spaces, most of which are non-symmetric. Taking the limit of an "inflating" sphere through a point $P$ in a Damek–Ricci space as the center of the sphere runs out to infinity along a geodesic half-line $\gamma$ starting from $P$, we get a horosphere. Similarly to spheres, horospheres are also isoparametric hypersurfaces. In this paper, we define and study the geometric properties of the sphere-like hypersurfaces obtained by "overinflating the horospheres" by pushing the center of the sphere beyond the point at infinity of $\gamma$ along a virtual prolongation of $\gamma$. We show for example, that these hypersurfaces are isoparametric hypersurfaces that can be obtained as tubes about a minimal submanifold of lower dimension.

## Compatibility conditions for three-dimensional unit vector fields

Luiz C. B. da Silva (joint work with E. Efrati)

Weizmann Institute of Science

Liquid crystals display phases with properties intermediate between fluids and crystals, e.g., cylinder-like molecules randomly placed in space but with axes aligned in the same direction. A unit vector field known as the *director* often describes the liquid crystalline ordering. The geometry and topology of the region where a director lives limit the kind of supported order, and these limitations manifest as compatibility conditions that relate the geometry of space to quantities describing the director. These quantities can be chosen as the coordinate-invariant components of the director's gradient and are often called *deformation modes*. As the deformation modes are obtained from derivatives of the director, their values are not independent, and the restrictions on them and their relation to space geometry are called *compatibility conditions*. So far, it was unknown how many local scalar fields are required to uniquely define a director, nor what compatibility relations they must satisfy. In Ref. [1], we addressed these questions directly by employing Cartan's method of moving frames. We showed that five deformation modes suffice to determine a director on a $3d$ Riemannian manifold, and they are related to each other and the curvature of space through six differential equations. As an application, we characterized *uniform distortion directors*, i.e., directors with constant deformation modes, in Riemannian manifolds of constant curvature. In Euclidean space, uniform directors are either constant or the unit velocity field of a foliation of space by parallel and congruent helices [1, 3]. On the other hand, in the hyperbolic space, they are the velocity field of foliations of space by non-parallel congruent helices [1]. In the sphere, they must be a Hopf fibration's velocity field [1]. Finally, we also use these compatibility conditions to characterize a wide collection of non-uniform compatible directors in Euclidean space [2].

[1] da Silva, L. C. B. and Efrati, E.: Moving frames and compatibility conditions for three-dimensional director fields. *New J. Phys.* **23** (2021) 063016.
[2] da Silva, L. C. B., Bar, T., and Efrati, E.: Compatible director fields in $\mathbb{R}^3$. *J. Elast.* (2023). https://doi.org/10.1007/s10659-023-09988-7.
[3] Virga, E. G.: Uniform distortions and generalized elasticity of liquid crystals. *Phys. Rev. E* **100** (2019) 052701.

## On the geometry of anti-quasi-Sasakian manifolds

Dario Di Pinto (joint work with G. Dileo)

Università degli Studi di Bari Aldo Moro

Introduced by D.E. Blair as a generalization of both Sasakian and cokähler manifolds, quasi-Sasakian manifolds are defined as normal almost contact metric manifolds $(M, \varphi, \xi, \eta, g)$, with closed fundamental $2$-form $\Phi$. The normality condition expresses the integrability of an almost complex structure on $M \times \mathbb{R}$ and it is equivalent to the vanishing of the tensor field $N_\varphi = [\varphi, \varphi] + d\eta \otimes \xi$. The Reeb vector field $\xi$ is Killing, the structure $(\varphi, g)$ is projectable along the $1$-dimensional foliation generated by $\xi$, and the transverse geometry is Kähler.

In the present talk I will introduce a new class of almost contact metric manifolds $(M, \varphi, \xi, \eta, g)$, defined in such a way that the structure $(\varphi, g)$ is still projectable, and the transverse geometry with respect to $\xi$ is given by a Kähler structure endowed with a closed $(2,0)$-form. Hyperkähler manifolds are well known examples of Kähler manifolds with a nondegenerate closed $(2,0)$-form. In order to introduce the new class, one needs to modify the normality condition into an *anti-normal* condition. Precisely, an *anti-quasi-Sasakian manifold* (aqS for short) is defined as an almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ such that

$$d\Phi = 0, \qquad N_\varphi = 2d\eta \otimes \xi.$$

Various examples of aqS manifolds can be provided, including compact nilmanifolds and $\mathbb{S}^1$-bundles. I will discuss general properties of aqS manifolds, focusing on the Riemannian curvature. In particular, aqS manifolds with constant $\xi$-sectional curvature equal to $1$ will be characterized: they admit a $Sp(n) \times 1$-reduction of the structural group of the frame bundle, such that the manifold is transversely hyperkähler, carrying a second aqS structure and a null Sasakian $\eta$-Einstein structure. I will also present some obstructions to the existence of aqS structures, one of them stating that aqS manifolds with constant sectional curvature are necessarily flat and cokähler.

## Classification of four-dimensional CR submanifolds of the homogenous nearly Kähler $\mathbf{S}^3 \times \mathbf{S}^3$ whose almost complex distribution is almost product orthogonal to itself

Nataša Djurdjević

Faculty of Agriculture, University of Belgrade

Butruille has shown that $\mathbf{S}^3 \times \mathbf{S}^3$ is one of the four homogeneous six-dimensional nearly Kähler manifolds. Besides its almost complex structure $J$, it also admits a canonical almost product structure $P$. The investigation of CR submanifolds of $\mathbf{S}^3 \times \mathbf{S}^3$ started recently. In this article an investigation of four-dimensional CR submanifolds is launched, because an investigation of some specific types of CR submanifolds leads to different types of submanifolds classified with respect to different positions of base vector fields under an action of the almost product structure $P$. The main result is the classification of four-dimensional CR submanifolds of $\mathbf{S}^3 \times \mathbf{S}^3$ whose almost complex distribution $\mathcal{D}_1$ is almost product orthogonal to itself. First, it is obtained that such submanifold $M$ has no integrable almost complex distribution $\mathcal{D}_1$ and and further it is proved that such submanifolds belong to the same type of CR submanifolds, whose almost complex distribution has an arbitrary vector field $E_1$ such as $PE_1 \in TM^\perp$. These submanifolds are locally product manifolds of curves and the three-dimensional CR submanifolds with a non-integrable almost complex distribution $\mathcal{D}_1$ for which $P\mathcal{D}_1 \perp \mathcal{D}_1$ holds, as well.

## The Bour's theorem for invariant surfaces in 3-manifolds

Iury Domingos (joint work with I. Onnis and P. Piu)

KU Leuven & Universidade Federal de Alagoas

The starting point of this talk is a classical result by Edmond Bour concerning helicoidal surfaces in Euclidean $3$-space. In 1862, Bour demonstrated the existence of a two-parameter family of helicoidal surfaces that are isometric to a given helicoidal surface in $\mathbb{R}^3$. Subsequently, Bour's theorem was extended to other ambient spaces, including certain homogeneous 3-manifolds. During this talk, we use techniques of equivariant geometry to establish that a generalized version of Bour's theorem holds for surfaces that remain invariant under the action of a one-parameter group of isometries in a 3-dimensional Riemannian manifold.

## Almost $\eta$-Ricci-Bourguignon solitons and integral formulas

Sibel Gerdan Aydin (joint work with M. Traore and H. M. Taştan)

KU Leuven & İstanbul University

In this study, we investigate the almost $\eta$-Ricci-Bourguignon soliton structure. We establish a relationship betweeen the potential vector field of a compact gradient almost $\eta$-Ricci-Bourguignon soliton and the Hodge-de Rham potential. We give some results when the potential vector field is a conformal vector field. We also show that the potential vector field of a compact almost $\eta$-Ricci-Bourguignon soliton is a Killing vector field under some conditions. Moreover, we obtain some integral formulas for the compact, orientable almost -$\eta$ Ricci-Bourguignon solitons.

## The H/Q-correspondence and a generalization of the supergravity c-map

Kazuyuki Hasegawa (joint work with V. Cortés)

Kanazawa University

Given a hypercomplex manifold $M$ with a rotating vector field and additional data, we construct a conical hypercomplex manifold $\hat{M}$ of $\dim \hat{M} = \dim M + 4$ with $\mathbb{H}^*(:= \mathbb{H}\backslash\{0\})$-action and show that $\bar{M} := \hat{M}/\mathbb{H}^*$ possesses a quaternionic structure. As a consequence, we associate the quaternionic manifold $\bar{M}$ to the hypercomplex manifold $M$ of the same dimension. This is a generalization of the HK/QK-correspondence ([1,3]). Here HK and QK mean hyperKähler and quaternionic Kähler, respectively. We call this construction $\bar{M}$ from $M$ the H/Q-correspondence. We can apply the H/Q-correspondence to a hypercomplex Hopf manifold which does not admit any hyperKähler structure. A compact Lie group $\mathrm{SU}(3)$ with the left invariant hypercomplex structure can be also applied to the H/Q-correspondence. These examples show that the H/Q-correspondence is a proper generalization of HK/QK-correspondence. As an application, we show that a quaternionic manifold can be associated to a conical special complex manifold of half its dimension. Furthermore, a projective special complex manifold (with a canonical c-projective structure) associates with a quaternionic manifold. The latter is a generalization of the supergravity c-map.

[1] D. Alekseevsky, V. Cortés and T. Mohaupt, Conification of Kähler and hyper-Kähler manifolds, Comm. Math. Phys. 324 (2013), 637–655.
[2] V. Cortés and K. Hasegawa, The H/Q-correspondence and a generalization of the supergravity c-map, Tohoku Math. J., to appear, arXiv:2207.09013.
[3] N. Hitchin, On the hyperkähler/quaternion Kähler correspondence, Comm. Math. Phys. 324 (2013), 77–106.

## Complete classification of local conservation laws for generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation

Pavel Holba

Silesian University in Opava

In this talk we consider nonlinear multidimensional Cahn–Hilliard and Kuramoto–Sivashinsky equations that have many important applications in physics, chemistry, and biology, and a certain natural generalization of these equations.

Namely, for an arbitrary natural number $n$ of spatial independent variables we consider the following PDE in $n+1$ independent variables $t, x_1, \ldots, x_n$ and one dependent variable $u$:

$$u_t = a\Delta^2 u + b(u)\Delta u + f(u)|\nabla u|^2 + g(u),$$

to which we refer as to the generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation as the Cahn–Hilliard equation and the Kuramoto–Sivashinsky equation are special cases thereof. Here $b, f, g$ are arbitrary smooth functions of the dependent variable $u$, $a$ is a nonzero constant, $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the Laplace operator and $|\nabla u|^2 = \sum_{i=1}^{n}(\partial u/\partial x_i)^2$.

We present a complete list of cases when the generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation admits nontrivial local conservation laws of any order, and for each of those cases we give an explicit form of all the local conservation laws of all orders modulo trivial ones admitted by the equation under study.

In particular, we show that the Kuramoto–Sivashinsky equation admits no nontrivial local conservation laws and list all of the nontrivial local conservation laws for the Cahn–Hilliard equation.

For more details, see [P. Holba, Complete classification of local conservation laws for generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation (https://doi.org/10.1111/ sapm.12576)].

## Approximating manifold-valued functions

Simon Jacobsson (joint work with R. Vandebril, J. Van der Veken and N. Vannieuwenhoven)

KU Leuven

Many functions that are worth approximating map into Riemannian manifolds. For example matrix and tensor decompositions. We present a construction using Riemann normal coordinates for approximating such functions. Our construc- tion extends approximation schemes for functions between linear spaces, like higher order SVD techniques and tensorized Chebyshev interpolation, in such a way that we are able to upper bound the max error in terms of a lower bound on the manifold's sectional curvature. Furthermore, when the sectional curvature is nonnegative, e.g. as for compact Lie groups, the error is guaranteed to be at least as good as in the linear case. Of special interest are manifolds that are not naturally embedded into a vector space or whose codimension is large. The Segre manifold of rank 1 tensors is an example of such a manifold where we are able to apply our construction. Approximating functions that map to the Segre manifold has applications in Model Order Reduction and tensor completion [1]. In some cases, the condition number of approximating functions that map into manifolds is exponential in the radius of the domain. We classify and discuss those cases.

[1] Swijsen, L. and Van der Veken, J. and Vannieuwenhoven, N. Tensor com- pletion using geodesics on Segre manifolds. Numerical Linear Algebra with Applications, 2022, 10.1002/nla.2446.

## Some remarks about Cotton solitons on almost $\alpha$-paracosymplectic manifolds

İrem Küpeli Erken (joint work with M. Özkan and B. Savur)

Bursa Technical University

In this paper, we study Cotton solitons on three-dimensional almost $\alpha$-paracosymplectic manifolds. We focus on three dimensional almost $\alpha$-paracosymplectic manifolds with harmonic vector field $\xi$ and characterize them for all possible types of operator $h$. We give the computations of the components of the $(0, 2)$-Cotton Tensor. Finally, an example was given which satisfies our results.

## Nearly Kähler $\mathbb{C}P^3$: from the Hopf fibration to Lagrangian submanifolds

Michaël Liefsoens (joint work with J. Van der Veken)

KU Leuven

We reintroduce the nearly Kähler homogeneous six-manifold $\mathbb{C}P^3$ by adapting the Fubini–Study metric of Kähler $\mathbb{C}P^3$ and twisting the standard complex structure. With this new and clear link between Kähler and nearly Kähler $\mathbb{C}P^3$, we are able to derive an explicit expression for the curvature tensor of nearly Kähler $\mathbb{C}P^3$, only in terms of elementary structures on the manifold. Moreover, by the relation to Kähler $\mathbb{C}P^3$ and the associated simple euclidean space, we are able to explicitly construct Lagrangian submanifolds and give further classification results.

## Einstein submanifolds with parallel mean curvature in product spaces

Fernando Manfio (joint work with E. Garcia)

University of São Paulo

In this work, we provide a classification of Einstein submanifolds in product spaces $\mathbb{Q}^n_\epsilon \times \mathbb{R}$, with flat normal bundle and parallel mean curvature vector field. This extends a previous result due to Leandro, Pina and dos Santos (Bull. Braz. Math. Soc., 2020) for Einstein hypersurfaces in such ambient space.

## Conformal dualities for prescribed mean curvature surfaces in Riemannian and Lorentzian three-manifolds

José Miguel Manzano (joint work with A. del Prete)

Universidad de Jaén

A well known result in minimal surface theory is the so-called Calabi duality between minimal graphs in Euclidean space $\mathbb{R}^3$ and maximal graphs in Lorentz-Minkoski space $\mathbb{L}^3$. Lee extended this result to constant mean curvature surfaces the Riemannian and Lorentzian homogeneous spaces $\mathbb{E}(\kappa, \tau)$ and $\mathbb{L}(\kappa, \tau)$ whose isometry is four-dimensional, but this property ultimately depends on the existence of a Killing vector field, as we shall discuss in this talk. We will also give two applications of the general duality. On the one hand, we will find entire graphs with prescribed mean curvature function in $\mathbb{L}^3$; on the other hand, we will investigate some connections between minimal surfaces in $\mathrm{Nil}_3$ and $\mathrm{Sol}_3$, the latter with isometry group of dimension three.

## Geometric inequalities on statistical submanifolds in Hessian manifolds

Ion Mihai

University of Bucharest

Statistical manifolds introduced, in 1985, by Amari have been studied in terms of information geometry. Since the geometry of such manifolds includes the notion of dual connections, also called conjugate connections in affine geometry, it is closely related to affine differential geometry. Further, a Hessian manifold is a statistical manifold.

On the other hand, curvature invariants are the main Riemannian invariants and the most natural ones. Curvature invariants also play key roles in physics.

The purpose of this talk is to state geometric inequalities on statistical submanifolds in Hessian manifolds of constant Hessian curvature.

## Rigidity of weighted Einstein manifolds under harmonicity conditions

Diego Mojón Álvarez (joint work with M. Brozos Vázquez)

CITMAga – Universidade de Santiago de Compostela

A Riemannian manifold $(M, g)$ can be endowed with a smooth density function $f$, and two parameters $m \in \mathbb{R}^+$ and $\mu \in \mathbb{R}$, giving rise to a smooth metric measure space $(M, g, f, m, \mu)$. In order to discuss smooth metric measure spaces, in addition to the well-known Bakry-Émery Ricci tensor, new weighted objects such as the weighted Schouten tensor $P_f^m$ or the weighted Weyl tensor $W_f^m$ have been introduced [2]. These tensors reflect different aspects of the influence of the density on the geometry of the underlying manifold.

Generalizing the concept of Einstein manifolds to this setting , weighted Einstein manifolds are smooth metric measure spaces which satisfy $P_f^m = \lambda g$ for some $\lambda \in \mathbb{R}$. Although Einstein metrics have harmonic Weyl tensor, this is no longer true if we introduce a non-constant density. Motivated by this fact, we aim to analyze which smooth metric measure spaces are weighted Einstein and have harmonic Weyl tensor (in a suitable weighted sense). To this end, we observe that, for weighted Einstein manifolds, both the density and the metric are real analytic in harmonic coordinates. We also present natural weighted analogues of the model spaces forms, and a complete local classification of our manifolds of interest is provided, showing that either the underlying manifold is Einstein, or decomposes as a warped product in a specific way. Moreover, if the manifold is complete, then we provide a global rigidity theorem which shows that it either is one of the aforementioned weighted space forms, or it belongs to a particular family of Einstein warped products with $\lambda < 0$.

[1] M. Brozos-Vázquez and D. Mojón-Álvarez; Rigidity of weighted Einstein smooth metric measure spaces. arXiv:2305.08516 [math.DG].
[2] J. S. Case; The weighted $\sigma_k$-curvature of a smooth metric measure space. *Pacific. J. Math.* **299** (2) (2019), 339–399.

## Magnetic Jacobi fields in almost contact metric manifolds

Marian Ioan Munteanu

Alexandru Ioan Cuza University of Iasi

The first variation of the Landau Hall functional on a Riemannian manifold leads to the notion of magnetic curves. Computing the second variation, we obtain the equation of a Jacobi-type field along a magnetic curve. In this talk we focus on the contact magnetic trajectories in Sasakian and cosymplectic manifolds (as ambient space) emphasising the main differences between the two cases. We will give several examples.

This talk is based on some joint papers:

1. M.I. Munteanu, *Magnetic geodesics in (almost) cosymplectic Lie groups of dimension 3*, Mathematics, (Special Issue: Topics in Differential Geometry), 10 (2022) 4, art. 544.

2. J. Inoguchi, M.I. Munteanu, *Magnetic Jacobi fields in 3-dimensional Sasakian space forms*, J. Geom. Analysis, 32 (2022) 3, art. 96.

3. M.I. Munteanu, A.I. Nistor, *Magnetic Jacobi fields in cosymplectic 3-dimensional manifolds*, Mathematics, (Special Issue: Differential Geometry: structures on manifolds and their applications), 9 (2021) 24 art. 3220.

4. J. Inoguchi, M.I. Munteanu, *Magnetic Jacobi fields in Sasakian space forms*, Mediterranean J. Mathematics, 20 (2023) art. 29.

## Classifying homogeneous hypersurfaces of symmetric spaces of noncompact type

Tomás Otero (joint work with J. C. Díaz-Ramos and M. Domínguez-Váquez)

CITMAga – Universidade de Santiago de Compostela

A hypersurface of a Riemannian manifold is said to be (extrinsically) homogeneous if it is an orbit of an isometric action on the ambient manifold. In this case, such action is said to be of cohomogeneity one. Symmetric spaces provide an excellent class of ambient spaces in which to study these actions: due to their rigid structure, it is possible to study homogeneous submanifolds in terms of the isometry Lie algebra of the symmetric space.

In this talk I will report on the state of the classification of cohomogeneity one actions on symmetric spaces, focusing on a joint work with J. Carlos Díaz-Ramos and Miguel Domínguez-Váquez where we propose an improved structural result for the classification of homogeneous hypersurfaces in symmetric spaces of noncompact type and arbitrary rank. This allowed us to produce the first classification result of homogeneous hypersurfaces in a space of rank >2, namely on the spaces $SL_n(\mathbb{R})/SO_n$.

## Pseudo-parallel and $\lambda$-isotropic submanifolds in semi-Riemannian manifolds

Oscar Palmas (joint work with G. Lobos, M. Melara and T. Kemajou-Mbiakop)

Universidad Nacional Autónoma de México

Pseudo-parallel immersions were introduced by Asperti-Lobos-Mercuri for Riemannian manifolds as a generalization of semi-parallel immersions, which in turn are a generalization of parallel immersions. We consider this notion when the ambient space is a semi-Riemannian manifold. In the case of an immersion of a timelike surface into a $4$-dimensional semi-Riemannian space form, the notion of pseudo-parallelism is equivalent to being $\lambda$-isotropic. In the second part of the talk we will discuss the notion of $\lambda$-isotropy for null submanifolds in Lorentzian space forms.

## Helix surfaces for Berger-like metrics on the anti-de Sitter space

Lorenzo Pellegrino (joint work with G. Calvaruso, I. I. Onnis and D. Uccheddu)

Università del Salento

A *helix surface* (or *constant angle surface*) is an oriented surface, whose normal vector field forms a constant angle with a fixed field of directions in the ambient space. In recent years many authors investigated helix surfaces in different Lorentzian ambient spaces, for example in the Minkowski space, the Lorentzian Heisenberg group and the Lorentzian Berger spheres. We focus on the Anti-de Sitter space $\mathbb{H}^3_1$ equipped with Berger-like metrics, that deform the standard metric of $\mathbb{H}^3_1$ in the direction of the hyperbolic Hopf vector field. These metrics, introduced by G. Calvaruso and D. Perrone, were induced in a natural way by corresponding metrics defined on the tangent sphere bundle $T_1\mathbb{H}^2(\kappa)$. We discuss and completely describe helix surfaces forming a constant angle with the hyperbolic Hopf vector field. After proving that these surfaces have (any) constant

Gaussian curvature, we achieve their explicit local description in terms of a one-parameter family of isometries of the space and some suitable curves. These curves turn out to be general helices, which meet at a constant angle the fibers of the hyperbolic Hopf fibration.

## On generalized Riemannian spaces in Eisenhart's sense and its applications in the calculus of variations

Miloš Z. Petrović

University of Nis

As is well-known generalized Riemannian spaces in Eisenhart's sense are differentiable manifolds endowed with a billinear form which is not necessarily symmetric. Obviously, such spaces include as particular cases Riemannian as well as pseudo-Riemannian and symplectic manifolds. Also, as it was pointed out in (Mileva Prvanović, Einstein connection of almost Hermitian manifold. Bull. Cl. Sci. Math. Nat. Sci. Math. 20, 51–59 (1995)) and (Miloš Z. Petrović, On generalized almost para-Hermitian spaces, XXI Geometrical Seminar, Belgrade, Serbia, June 26 – July 2, 2022.) special cases of these spaces are almost Hermitian manifolds as well as almost para-Hermitian manifolds. We give some examples of generalized Riemannian spaces in Eisenhart's sense and provide some applications of these spaces in the calculus of variations.

## Breakdown of the EPDiff equation

Stephen Preston

Brooklyn College and CUNY Graduate Center

The EPDiff equation is the PDE describing geodesics on the group of diffeomorphisms of an $n$-dimensional manifold, when equipped with a right-invariant Sobolev metric. It is important in the study of pattern recognition and shape analysis, and also appears in some fluid mechanical models. In this talk I will describe some recent work on how to prove that solutions of the equation break down in finite time, using a new method of comparison theory of infinite-dimensional ODEs. The main result is that solutions with the $H^2$ metric break down on $\mathbb{R}^n$ when $n \geq 3$ but exist globally when $n = 1$ or $n = 2$; however the method can be applied much more generally.

## Totally geodesic submanifolds in homogeneous spheres

Alberto Rodríguez-Vázquez

KU Leuven

The classification of transitive Lie group actions on spheres was obtained by Borel, Montgomery, and Samelson in the forties. As a consequence of this, it turns out that apart from the round metric there are other Riemannian metrics in spheres which are invariant under the action of a transitive Lie group. These other homogeneous metrics in spheres can be constructed by modifying the metric of the total space of the complex, quaternionic or octonionic Hopf fibration in the direction of the fibers. In this talk, I will report on an ongoing joint work with Carlos Olmos (Universidad Nacional de Córdoba), where we classify totally geodesic submanifolds in those Riemannian homogeneous spheres obtained by rescaling the round metric of the total space of Hopf fibrations by a positive factor in the direction of the fibers.

## Genus one $H$-surfaces with $k$ ends in $\mathbb{H}^2 \times \mathbb{R}$

José S. Santiago (joint work with J. Castro-Infantes)

University of Jaén

In this talk we will construct via Daniel's sister correspondence in $\mathbb{H}^2 \times \mathbb{R}$ two families of properly Alexandrov-immersed surfaces with constant mean curvature $0 < H < 1/2$ in $\mathbb{H}^2 \times \mathbb{R}$ with genus one and $k$ ends asymptotic to vertical $H$-cylinders. Moreover, we will show the existence of properly Alexandrov-immersed $H$ surfaces with genus one and $2$ ends each of them asymptotic to a vertical cylinder from the convex side displaying that there is not a Schoen-type theorem for immersed $H$-surfaces in $\mathbb{H}^2 \times \mathbb{R}$.

## The first eigenvalue of the Laplacian

Shoo Seto

California State University, Fullerton

In this talk, we will survey some results on the first eigenvalue of the Laplacian on Riemannian manifolds under a Ricci curvature lower bound and its generalization to integral Ricci curvature.

## On multidimensional affine submanifolds of codimension two

Olena Shugailo

V. N. Karazin Kharkiv National University

We consider the affine immersions by K. Nomizu, T. Sasaki [3], namely $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$. For a transversal frame $\xi_1,\ \xi_2$ and tangent vector fields $X,\ Y$ we have the affine analogues of Gauss and Weingarten decompositions, namely

$$D_X f_*(Y) = f_*(\nabla_X Y) + h^\alpha(X,\ Y)\xi_\alpha,$$

$$D_X \xi_\alpha = -f_*(S_\alpha X) + \tau_\alpha^\beta(X)\xi_\beta,$$

where $h^\alpha$ are components of the affine fundamental form, $S_\alpha$ are shape operators, $\tau_\alpha^\beta$ are forms of transversal connection (with respect to $\xi_1,\ \xi_2$).

The *Weingarten mapping* $S_x : Q_x \times T_x(M^n) \to T_x(M^n)$ is defined [5] as follows: $(\xi, X) \mapsto S_\xi X$ at every point $x \in M^n$ (where $T_x(M^n)$ and $Q_x$ are tangent and transversal distributions.)

We consider an affine immersion $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$ with some additional requirements for connection $\nabla$, Weingarten mapping, affine fundamental form or and transversal distribution.

Two-codimensional affine surfaces with different additional properties have been studied by many authors. Flat affine surfaces in $\mathbb{R}^4$ with flat normal connection were studied in [1]. The description of a parallel affine immersions $(M^n, \nabla) \to \mathbb{R}^{n+2}$ with flat connection in dependence on the rank of the Weingarten mapping were given in [5].

It was proved [7] that for the immersion $f : (M^n, \nabla) \to \mathbb{R}^{n+2}$ with maximal rank of affine fundamental form and flat connection $\nabla$ the following relations hold true:

$$1)\, \dim\ker S \geq n-2; \qquad 2)\, \ker h \subseteq \ker S; \qquad 3)\, \dim \operatorname{im} S \leq 2;$$

$$4)\text{ if } \dim \operatorname{im} S = 2, \text{ then } \dim\ker S = n-2 \text{ and } \ker h = \ker S.$$

So we have only three possible values for the dimension of $\operatorname{im} S$, namely $0,\ 1,\ 2$. The most studied are affine immersions with zero and two-dimensional Weingarten mapping.

It is well known that an affine immersion with zero Weingarten mapping ($S \equiv 0$) has a flat connection and it is affinely equivalent to the graph of certain smooth map $F : M^n \to \mathbb{R}^2$ (see for example [2,3,7]). Obviously, a graph immersion has a flat connection and it is equiaffine one.

According to the properties which were discussed in [7], in case $\dim \operatorname{im} S = 2$ we obtain $\ker h = \ker S$ and $\dim\ker h = n-2$. Therefore such a submanifold is a submanifold of rank two (by the rank of Gaussian (Grassmann) mapping) [6]. Rank-two submanifold is a ruled submanifold with $(n-2)$-dimensional rulings over a two-dimensional base. In case this submanifold is a cylinder, its connection is determined by the connection of the cylinder base. In the general case the problem on its connection remains open.

We obtain a parametrization of a submanifold with rank two affine fundamental form, equiaffine structure, flat connection $\nabla$, and one-dimensional Weingarten mapping:

(i) $\vec{r} = g(u^1, \dots, u^n)\vec{a}_1 + \int \vec{\varphi}(u^1)du^1 + \sum\limits_{i=2}^{n} u^i \vec{a}_i;$

(ii) $\vec{r} = (g(u^2, \dots, u^n) + u^1)\vec{a} + \int v(u^1)\vec{\eta}(u^1)du^1 + \sum\limits_{i=2}^{n} u^i \int \lambda_i(u^1)\vec{\eta}(u^1)du^1;$

(iii) $\vec{r} = (g(u^2, \dots, u^n) + u^1)\vec{\rho}(u^1) + \int (v(u^1) - u^1)\dfrac{d\vec{\rho}(u^1)}{du^1}du^1 + \sum\limits_{i=2}^{n} u^i \int \lambda_i(u^1)\dfrac{d\vec{\rho}(u^1)}{du^1}du^1.$

Such a submanifold is a peculiar "mix" of a graph and a ruled submanifold.


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## Geometrical eigenproblem of higher order tensors on Riemannian manifolds

Jelena Stojanov (joint work with V. Balan)

University of Novi Sad

Extensions of the classical eigenproblem toward tensors have two directions based on different considerations of tensor as a multidimensional data array, or as a multilinear operator. All the extensions have found wide range of applications, but none of them possess both the main properties of the classical eigenproblem: the homogeneity and the invariance.

Recently, geometrical eigenproblem (G-eigenproblem) of covariant supersymmetric tensor field on Riemannian manifold has been promoted. The G-eigenproblem is an adjustment of the well-known Z-eigenproblem. The G-eigenproblem is invariant of local representation and at least positively homogeneous and it has eigenspaces. The main features and benefits of the G-eigenproblem, as well as the drawbacks, will be exposed. Its relation with the Z-eigenproblem will be discussed. Examples will be presented.

G-eigenproblem of contravariant and mixed tensor fields will be considered and defined in proper way to keep full geometrical meaning, and to indicate possible application directions.

## On spacelike surfaces in Robertson-Walker space-times

Nurettin Cenk Turgay (joint work with B. Bektaş Demirci and R. Yeğin Şen)

Istanbul Technical University

In this work, we study space-like surfaces in the Robertson-Walker space time $L_1^4(f,0) = (I \times \mathbb{R}^3, \tilde{g})$ for a non-vanishing smooth function $f$, where $I$ is an open interval,

$$\tilde{g} = -dt^2 + f(t)^2 g_0$$

and $g_0$ denotes the metric tensor of the Euclidean space $\mathbb{E}^3$.

Note that for a given surface $M$ in $L_1^4(f,0)$, one can define a tangent vector field $T$ and a normal vector field $\eta$ by splitting co-moving observer field $\partial_t$ by the equation

$$\partial_t = T + \eta.$$

We first construct a special local parametrization on $M$. Then, we obtain several classification of space-like surfaces which satisfy certain geometrical conditions in terms of the vector fields $T$ and $\eta$. Finally, we get results on surfaces in the arbitrary dimensional space $L_1^n(f,0)$.

## A local geometrical, metrical solution of Thurston's geometrical space form problem

Pol Verstraelen

KU Leuven

A proposal is made for what may well be the most elementary Riemannian spaces which are homogeneous but not isotropic. In other words: a proposal is made for what may well be the nicest symmetric spaces beyond the real space forms, that is beyond the Riemannian spaces which are homogeneous and isotropic. The qualification of 'nicest symmetric spaces' may find a justification in that these spaces are most natural with respect to the importance in human vision of the ability to readily recognise similar things and in that these spaces are most natural with respect to Weyl's view on what is symmetry in Riemannian geometry.

## Some notes on a quadratic functional of the $\varphi$-scalar curvature

Handan Yıldırım (joint work with M. Rigoli)

Istanbul University

Let $(M, g)$, $m \geq 2$ and $\left(N, \langle\ ,\ \rangle_N\right)$ be two Riemannian manifolds and $\varphi : M \to N$ be a smooth map. The $\varphi$-Ricci tensor $Ric^{\varphi}$ of $M$ is defined by

$$Ric^{\varphi} = Ric - \alpha\varphi^{*}\langle\ ,\ \rangle_N,$$

where $\alpha \in \mathbb{R} \setminus \{0\}$ is a coupling parameter (See [3, 4, 5] for the details). Thus, $\varphi$-scalar curvature $S^{\varphi}$ of $M$ is defined to be the trace of $Ric^{\varphi}$ with respect to the metric $g$.

In this talk which is based on a joint work with Marco Rigoli, in terms of [2], we first deduce the Euler-Lagrange equations of the following quadratic action functional

$$A_{\Omega}[g, \varphi] = \int_{\Omega} (S^{\varphi})^2 dV_g$$

with respect to compactly supported variations of $g$ and $\varphi$, where $\Omega$ is a relatively compact domain in $M$. These equations give rise to a particular Einstein-type structure on $M$ as defined in [3]. Next, taking into account this together with the completeness of $g$ and two more mild assumptions, by means of [1], we obtain that $M$ is $\varphi$-scalar flat when it is of at least dimension $5$. We emphasize that this result is also new when $\varphi$ is constant.

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# Posters

## Parallel and totally umbilical hypersurfaces of the Thurston geometry $\mathrm{Sol}_0^4$

Marie D'haene (joint work with J. Inoguchi and J. Van der Veken)

KU Leuven

Located on the crossroads of geometry, algebra and group theory, Thurston geometries are a fundamental class of manifolds to study. The 3-dimensional ones, $\mathbb{R}^3$, $\mathbb{S}^3$, $\mathbb{H}^3$, $\mathbb{S}^2 \times \mathbb{R}$, $\mathbb{H}^2 \times \mathbb{R}$, $\widetilde{\mathrm{SL}}(2,\mathbb{R})$, $\mathrm{Nil}^3$ and $\mathrm{Sol}^3$, are the model spaces in Thurston's geometrization for 3-manifolds. This is a deep result listing the possible Riemannian structures of compact orientable 3-manifolds, analogous to the uniformization of surfaces. Up until today, no such geometrization is known in dimension 4, however the study of 4-dimensional Thurston geometries is interesting on its own from a Riemannian geometric point of view. Here, we focus on a four-dimensional Thurston geometry, $\mathrm{Sol}_0^4$, and some of its special submanifolds. This space is a matrix Lie group and its isometry group is $\mathrm{Sol}_0^4 \rtimes (\mathrm{O}(2) \times \mathbb{Z}/2\mathbb{Z})$, hence it enjoys many symmetries.

The main result of the presented research is a *full classification of totally umbilical hypersurfaces* and of hypersurfaces for which the *second fundamental form is a Codazzi tensor* (this class includes parallel hypersurfaces). In addition, we show a neat expression for the Riemann curvature tensor of $\mathrm{Sol}_0^4$ and discuss some general properties of the space.

## On minimal CR-submanifolds in $\mathrm{Sol}_0^4$

Zlatko Erjavec (joint work with J. Inoguchi)

University of Zagreb

In this overview we present results on minimal invariant, minimal totally real and minimal CR-submanifolds in $\mathrm{Sol}_0^4$ model space equipped with standard globally conformal Kähler structure.

The model space $\mathrm{Sol}_0^4$ is one of the nineteen $4$-dim Thurston geometries belonging to the non-Kähler class of model spaces.

It is known that the Hermitian structures of all non-Kähler model spaces are locally conformal Kähler. A *CR-submanifold* $M$ is a Hermitian submanifold endowed with a distribution $\mathfrak{D}$ such that $\mathfrak{D}$ is *invariant* (or *holomorphic*), i.e. $J_p\mathfrak{D}_p = \mathfrak{D}_p$, and the orthogonal complement $\mathfrak{D}^\perp$ is *anti-invariant* (or *totally real*), i.e. $J_p\mathfrak{D}_p^\perp \subseteq T_p^\perp M,\ \forall p \in M,$ where $T_p^\perp M$ is the normal space at $p$.

Special cases of CR-submanifolds are invariant and totally real surfaces. We prove that the only minimal invariant surfaces of $\mathrm{Sol}_0^4$ are totally geodesic hyperbolic planes. Next, we prove that minimal totally real surfaces in $\mathrm{Sol}_0^4$ which are tangent or normal to the Lee field are cylindrical surfaces.

Finally, we consider proper minimal CR-submanifolds. We present some minimal CR-product, minimal CR-warped product and minimal warped product CR-submanifolds.

## The $\kappa$-nullity of Riemannian manifolds and their splitting tensors

Felippe Guimarães (joint work with C. Gorodski)

KU Leuven

We consider Riemannian $n$-manifolds $M$ with nontrivial $\kappa$-nullity “distribution" of the curvature tensor $R$, namely, the variable rank distribution of tangent subspaces to $M$ where $R$ coincides with the curvature tensor of a space of constant curvature $\kappa$ ($\kappa \in \mathbb{R}$) is nontrivial. We obtain classification theorems under diferent additional assumptions, in terms of low nullity/conullity, controlled scalar curvature or existence of quotients of finite volume. We prove new results, but also revisit previous ones.

## Geometric inequalities for isotropic submanifolds in pseudo-Riemannian space forms

Marius Mirea

University of Bucharest

The class of isotropic submanifolds in pseudo-Riemannian manifolds is a distinguished family of submanifolds; they have been studied by several authors. We establish a generalized Euler inequality and a Ricci inequality for spacelike submanifolds of pseudo-Riemannian space forms. Then we establish several Chen inequalities for isotropic immersions. An example of an isotropic immersion for which the equality case in the Chen first inequality holds is given.

## Some results about Cotton solitons on paracontact metric manifolds

Mustafa Özkan (joint work with İ. Küpeli Erken and C. Murathan)

Bursa Technical University

In this paper, we study Cotton solitons on three-dimensional paracontact metric manifolds. We especially focus on three-dimensional paracontact metric manifolds with harmonic vector field $\xi$ and characterize them for all possible types of operator h. Finally, we constructed an example which satisfies our results.

## On the biconservative rotational surfaces in Minkowski 4-space

Hazal Yürük (joint work with N. C. Turgay and R. Yeğin Şen)

Istanbul Technical University

An isometric immersion $F : (M, g) \to (N, \tilde{g})$ is said to be biharmonic if it is a criticial point of the bi-energy functional

$$E_2(F) = \int_M \|\tau(F)\|^2 v_g,$$

where $\tau(F) := \mathrm{trace}\nabla F_*$ is the tension field and $v_g$ is the volume element of $M$. On the other hand, if F satisfies the weaker condition

$$\langle \tau_2(F), F_* \rangle = 0$$

then F is said to be biconservative, where

$$\tau_2(F) := -\Delta\tau(F) - \mathrm{trace}R^N(F_*, \tau(F))F_* = 0$$

and $R^N$ is the curvature tensor of $(N, \tilde{g})$.

In this work, we study rotational surfaces 4-space $E^4_1$ given by

$$F_1(s,t) = (x(s), y(s), w(s)\sinh t, w(s)\cosh t),$$

$$F_2(s,t) = (x(s), y(s), w(s)\cosh t, w(s)\sinh t)$$

and

$$F_3(s,t) = \left( x(s), \sqrt{2}tw(s), \frac{(-z(s) - t^2w(s) + w(s))}{\sqrt{2}}, \frac{(z(s) + t^2w(s) + w(s))}{\sqrt{2}} \right)$$

for some smooth functions $x, y$ and $w$. We obtain the complete classification of biconservative rotational surfaces.

# Schedule

## Monday, July 10th

| | | |
|---|---|---|
| 9:30–11:15 | **Registration** | |
| 11:15–11:30 | **Opening session** | |
| 11:30–12:30 | **Keti Tenenblat** | |
| 12:30–14:00 | **Lunch** | |
| 14:00–14:30 | José Santiago | Cornelia-Livia Bejan |
| 14:30–15:00 | Iury Domingos | Dario Di Pinto |
| 15:00–15:30 | Lorenzo Pellegrino | Irem Küpeli Erken |
| 15:30–16:00 | **Coffee break** | |
| 16:00–17:00 | **Bruno Premoselli** | |

## Tuesday, July 11th

| | | |
|---|---|---|
| 09:30–10:30 | **Anna Siffert** | |
| 10:30–11:00 | **Coffee break** | |
| 11:00–11:30 | Nurettin Cenk Turgay | Ángel Cidre-Diáz |
| 11:30–12:00 | Oscar Palmas | Burcu Bektaş Demirci |
| 12:00–12:30 | Balázs Csikós | Sibel Gerdan Aydin |
| 12:30–14:00 | **Lunch** | |
| 14:00–14:30 | Fernando Manfio | Diego Mojón Álvarez |
| 14:30–15:00 | Hiba Bibi | Handan Yıldırım |
| 15:00–15:30 | Stephen Preston | Zeina Al Dawoud |
| 15:30–16:00 | **Coffee break** | |
| 16:00–17:00 | **Barbara Tumpach** | |

## Wednesday, July 12th

| | |
|---|---|
| 09:30–10:30 | **Bogdan Suceava** |
| 10:30–11:30 | **Coffee break and Poster session** |
| 11:00–11:30 | Alberto Rodríguez-Vázquez |
| 11:00–11:30 | Ion Mihai |
| 11:30–12:00 | Pol Verstraelen |
| 12:30–14:00 | **Lunch** |
| 14:00 | **Free afternoon** |

## Thursday, July 13th

| | | |
|---|---|---|
| 09:30–10:30 | **Miguel Domínguez Vázquez** | |
| 10:30–11:00 | **Coffee break** | |
| 11:00–11:30 | Shoo Seto | Olivier Brahic |
| 11:30–12:00 | Pavel Holba | Tomás Otero |
| 12:00–12:30 | Jelena Stojanov | Luiz da Silva |
| 12:30–14:00 | **Lunch** | |
| 14:00–14:30 | Miloš Petrović | Mateo Anarella |
| 14:30–15:00 | Olena Shugailo | Michaël Liefsoens |
| 15:00–15:30 | Simon Jacobsson | Nataša Djurdjević |
| 15:30–16:00 | **Coffee break** | |
| 16:00–17:00 | **Francisco Torralbo** | |
| 19:00 | **Conference dinner** | |

## Friday, July 14th

| | |
|---|---|
| 09:30–10:30 | **Benoît Daniel** |
| 10:30–11:00 | **Coffee break** |
| 11:00–11:30 | Marian Ioan Munteanu |
| 11:30–12:00 | Kazuyuki Hasegawa |
| 12:00–12:30 | José Miguel Manzano |
| 12:30–14:00 | **Lunch** |
| 14:00–15:00 | **Ruy Tojeiro** |
| 15:00–15:15 | **Closing session** |

# Committees and participants

## Scientific committee

Bang-Yen Chen (Michigan State University)

Simone Gutt (Université Libre de Bruxelles)

Haizhong Li (Tsinghua University)

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