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A CHAOTIC GROWTH MODEL OF AGRICULTURAL POPULATION

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Abstract: Using the autoregression models, the paper considers movement of agricultural population. Irregular movement of agricultural population can be analyzed within the formal framework of the chaotic growth model.

The basic aims of this paper are: firstly, to set up a chaotic growth model of agricultural population; and secondly, to analyze the stability of agricultural population movement according to the presented logistic growth model in the world and eight group of countries in the period 1967-1997.

Key words: agricultural population, rate of growth, stability.

Introduction

There is in prospect a rather constant growth of the world agricultural population. However, the growth rate of agricultural population points to unstable relative changes of this category of population.

The growth rate of agricultural population peaked in the second half of the 1960s at 1.35 percent and had fallen by the late 1970s. Further, the growth rate of agricultural population increased in the 1980s and had fallen to 0.4 percent by the late 1990s.

In this paper agricultural population is divided according to the classification of countries and territories used by FAO (FAO, 2001). Differences in natural conditions and degrees of economic development generate different tendencies of agricultural population growth rate movement. However, all analyzed classes of countries are characterised by nonperiodic cycles. Nonperiodic cycles tend to appear in nonlinear systems, which are inherently upredictable in the long run. Namely, chaotic, complex or irregular systems do not follow any traditional patterns such as monotonic or periodic convergence or divergence. Its time-series

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appears to be erratic, although the system is completely deterministic and no random factors are present.

This paper analyzes an irregular movement of agricultural economically active population within the formal framework of logistic model . The analysis is oriented toward comparison of chaotic growth models. These models are estimated on empirical data of analyzed classes of countries during the period 1966-1999.

The model

Irregular movement of agricultural population can be analyzed within the formal framework of the chaotic growth model.

The growth rate of agricultural economically active population can be presented as

$$\frac{Y_{n+1} - Y_n}{Y_n} = \alpha \qquad (1)$$

where Y_n denotes agricultural population in n, and Y_{n+1} denotes agricultural population in n+1. The growth model assumes that the growth rate of agricultural population , α , is constant.

The growth law is

$$Y_{n+1} = Y_t + \alpha Y_n = (1+\alpha)\, Y_n$$
or
$$Y_n = (1+\alpha)^n \; Y_0 \qquad (2)$$

where Y_0 is the initial size of agricultural population with which we start our observations at time 0. In other words, knowing α and measuring Y_0 would suffice to predict Y_n for any point in time. The most simple growth model , linear model, would assume a constant growth rate of agricultural population, but in that situation we find unlimited growth which is not realistic.

Further, it is assumed that particular value of agricultural population in time series is restricted by its highest value in observed time series. This premise requires a modification of the population growth law. Now, the growth rate depends on the acutal size of agricultural population , Y_n , relative to its maximal size Y^m. We introduce y as $y= Y/Y^m$. Thus y ranges between 0 and 1, i.e. we can interpret y=0.85 as the actual size of agricultural population , Y, being 85% of its maximal size Y^m.

Again we index y by n, i.e., write y_n to refer to the size at time steps n =0,1,2,3,... Now the growth rate of agricultural population is measured by the quantity

$$\frac{y_{n+1} - y_n}{y_n} = \alpha \qquad (3)$$

Further, it is supposed that the growth rate of agricultural population is directly proportional to the gap between maximal and particular size of agricultural population in its time series:

$$\frac{y_{n+1} - y_n}{y_n} \text{ " directly proportional to " } 1 - \beta y_n$$

or, after introducing a suitable constant α

$$\frac{y_{n+1} - y_n}{y_n} = \alpha (1 - \beta y_n) \qquad (4)$$

Solving the last equation yields the growth model

$$y_{n+1} = y_n + \alpha y_n (1 - \beta y_n) \qquad (5)$$

or

$$y_{n+1} = (1+ \alpha) y_n - \alpha \beta y_n^2 \qquad (6)$$

Consider the growth model (6) the steady-state points, y^r , are obtained by solving:

$$y^r = y^r + \alpha y^s - \alpha\beta y^{r\,2} = 0$$

or

$$y^r - y^r + \alpha y^s - \alpha\beta y^{r\,2} = 0.$$

Simplifying gives

$$y^r (- \alpha + \alpha\beta y^r) = 0.$$

The two steady-state points are : $y^r = 0$ i $y^r = \frac{1}{\beta}$.

To determine the stability properties of these two equilibria, we apply theorem which requires that we evaluate the derivative of equation (6) at the points y^r.

Namely, the derivative of equation (6)

$$\frac{d\,y_{n+1}}{d\,y_n} = 1+\alpha - 2\,\alpha\beta \quad \begin{cases} 1+\alpha & \text{in } y^r = 0 \\ 1-\alpha & \text{in } y^r = \dfrac{1}{\beta} \end{cases}$$

The point $y^r = \frac{1}{\beta}$ is locally stable only if $|1 - \alpha| < 1$. Expressed differently, the positive steady-state point is locally stable if and only if $0 < \alpha < 2$.

Empirical evidence

The main aim of this paper is to analyze the stability of movement of agricultural population in the world and eight groups of countries over the period 1966-1999 by using the presented non-linear, logistic growth model (6).

Firstly, we transform data (FAO, 1998) on the agricultural population , Y_n , from 0 to 1, according to our supposition that actual size of the agricultural population , Y_n , is restricted by its highest value in the time-series, Y^m. Further , we obtain time-series of $y = Y_n / Y^m$. Now, we estimate the first degree autoregression models of the model (6) by the method of the minimal squares. The results are presented in Table 1.

Tab. 1. - Estimated logistic growth model of agricultural population (6)

Group of countries	Estimated autoregression models	Index of correlation	Coefficient of variation
World	$y_{n+1} = 1.0396\ y_n - 0.0344\ y_n^2$	1.0000	0.12%
European Union	$y_{n+1} = 0.9658\ y_n - 0.00002\ y_n^2$	1.0000	0.56%
Developed countries	$y_{n+1} = 0.9846\ y_n - 0.0095\ y_n^2$	1.0000	0.62%
Developing countries	$y_{n+1} = 1.0449\ y_n - 0.0387\ y_n^2$	0.9999	0.12%
Countries in transition	$y_{n+1} = 0.9847\ y_n - 0.0019\ y_n^2$	0.9982	0.85%
Sub-Saharan Africa	$y_{n+1} = 1.0272\ y_n - 0.0091\ y_n^2$	0.9999	0.11%
Near East and North Africa	$y_{n+1} = 1.0813\ y_n - 0.0811\ y_n^2$	0.9933	0.41%
Asia and Pacific / Far East and Oceania	$y_{n+1} = 1.0512\ y_n - 0.0457\ y_n^2$	0.9999	0.16%
Latin America and the Caribbean	$y_{n+1} = 0.9537\ y_n + 0.0469\ y_n^2$	0.9882	0.68%

The coefficient α ranges between 0 and 2 in the autoregression models in the world, Asia and Pacific / Far East and Oceania, developing countries, Near East and North Africa, Sub-Saharan Africa. These groups of countries are characterized by local stability of the positive steady-state points. Their values are

1.1511; 1.1197; 1.1597; 1.0019 and 2.9976, respectively. These groups of countries are characterized by steady growth of absolute size of agricultural population.

It is established that negative values of the coefficient α and absence of the positive equilibrium value should be associated to the European Union, developed countries and countries in transition. Simultaneously, these countries are characterized by the tendency of permanent diminution of absolute size of agricultural population.

In Latin America and the Caribbean agricultural population increased in1966 -80. Latin America and the Caribbean experienced a fall in agricultural population after 1980. This tendency determines the negative value of the coefficient α , and the positive value of steady-state point, 1.0130.

Index of correlation and coefficient of variation of autoregression are established as a measure of representativity of estimated equations. High values of the index of correlation exhibits almost deterministic relation between the successive sizes of agricultural population in the analyzed groups of countries. Selected functional relation is valid because the values of the coefficients of variation range below 1%.

Conclusion

Analysis of time-series can be focused to examine factors which determine the course of phenomenon represented by time-series or to study degree and form of interdependence between members of time-series according to time lags. For many economic phomena it is possible to analyze the form of relation in successive periods or in many separated periods, i.e. autoregression.

Function of autoregression could have different forms. This paper has used the logistic model to analyze the stability of agricultural population in the world and the selected groups of countries in the1966-99.period. A key hypothesis of this work is based on the idea that the coefficient α plays a crucial role in explaining the stability of agricultural population movement.

Validity of the analyzed growth model is confirmed through different values of the coefficient α and existence of positive steady-state points in accordance with the direction of agricultural population movement in the observed period.

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HAOTIČAN MODEL RASTA POLJOPRIVREDNOG STANOVNIŠTVA

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Rezime

U svetu, apsolutni nivo poljoprivrednog stanovništva je u stalnom porastu. Medjutim, stope rasta poljoprivrednog stanovništva ukazuju na nestabilnost relativnih promena ove kategorije stanovništva.

Stopa rasta poljoprivrednog stanovništva je dostigla maksimum krajem 60-tih godina, kada je iznosila 1,35%. U toku 70-tih godina je padala, u toku 80-tih rasla, da bi ponovo padala u toku 90-tih godina , a poslednjih godina se kretala oko 0,4% .

Osnovni cilj ovog rada je: prvo, postaviti haotičan model rasta poljoprivrednog stanovništva; i drugo, prema prezentiranom logističkom modelu rasta analizirati stabilnost kretanja poljoprivrednog stanovništva u svetu i u osam grupa zemalja u periodu od 1966. do 1999.godine.

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Koeficijent α ima vrednost iz intervala koji zadovoljava uslov lokalne stabilnosti ravnoteže, tj., izmedju 0 i 2 u autoregresionim modelima za svet, Aziju i Pacifik, zemlje u razvoju, Bliski Istok i Severnu Afriku, kao i Sub-Saharsku Afriku. Ove grupe zemalja karakteriše lokalno stabilne pozitivne ravnotežne tačke, koje respektivno iznose: 1,1511; 1,1197; 1,1597; 1,0019 i 2,9976, a takodje i stalni rast apsolutnog nivoa poljoprivrednog stanovništva.

Za zemlje Evropske Unije, razvijene zemlje i zemlje u tranziciji je utvrdjen negativan koeficijent α i odsustvo pozitivne tačke ravnoteže. To su ujedno i grupe zemalja u kojima je prisutna tendencija stalnog smanjenja poljoprivrednog stanovništva u apsolutnom iznosu.

Zemlje Latinske Amerike i Karipskih ostrva se izdvajaju od ostalih grupa zemalja. Naime, u njima broj poljoprivrednog stanovništva je bio u porastu od 1966. do 1980. godine, a posle toga je počeo da opada. Ova specifičnost se odrazila tako da koeficijent α ima negativnu vrednost, ali postoji tačka ravnoteže, 1, 0130.

Reprezentativnost analiziranog modela je potvrdjena vrednostima indeksa korelacije oko 1 i koeficijentima varijacije nižim od 1 %.

Validnost razmatranog modela rasta potvrdjena je kroz različite vrednosti koeficijenta α , i prisustvo ili odsustvo pozitivnih ravnotežnih tačaka u skladu sa smerom kretanja poljoprivrednog stanovništva u posmatranom periodu.

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