Available online at www.sciencedirect.com

ScienceDirect

Procedia Technology 8 (2013) 130 – 133

Procedia Technology

6th International Conference on Information and Communication Technologies in Agriculture, Food and Environment (HAICTA 2013)

The Chaotic Price Growth Model of the Agricultural Monopoly and New Information and Communication Technology

Vesna D. Jablanovic[a]*

[a]*University of Belgrade, Faculty of Agriculture, Nemanjina 6, Belgrade-Zemun ,11081, Serbia*

Abstract

Patent is example of how the government creates an agricultural monopoly to serve the public interest. Patent law leads to higher prices than would occur under competition. But by allowing these agricultural monopoly producers to charge higher prices and earn higher profits, the patent laws also encourage research by these agricultural monopoly firms. Agricultural monopoly has the ability to influence the market price. Except an agricultural monopolist's choice of price and output, there are other decisions an agricultural monopolist must make. One of the most important is how much to invest in new information and communication technology. Agricultural monopoly will invest in a new information and communication technology whenever doing so lowers its costs. The basic aim of this paper is to construct a relatively simple chaotic growth model of the agricultural government-created monopoly price that is capable of generating stable equilibria, cycles, or chaos. Incentives to invest in a new information and communication technology are included in model.

Keywords: Agricultural Monopoly; Price; Stability; New Information and Communication Technology Chaos

1. Introduction

Chaos theory started with Lorenz's [14] discovery of complex dynamics arising from three nonlinear differential

* Corresponding author. Tel.:+381-11-2615-315; fax:+381-11-3161-730.
*E-mail address:*vesnajab@ptt.rs

doi:10.1016/j.protcy.2013.11.018

equations leading to turbulence in the weather system. Li and Yorke [13] discovered that the simple logistic curve can exibit very complex behaviour. Further, May [16] described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day [1, 2], Day [4, 5, 6], Grandmont [8], Goodwin [7], Medio [17], Lorenz [15], Jablanovic [9, 10, 11, 12], among many others.The basic aim of this paper is to provide a relatively simple chaotic growth model of the agricultural government- created monopoly's price that is capable of generating stable equilibria, cycles, or chaos. Incentives to invest in a new information and communication technology are included in model.

2. A Simple Chaotic Agricultural Monopoly Price Growth Model

In the model of a profit-maximizing agricultural monopoly, take the inverse demand function

$$P_t = n - m\,Q_t \qquad n > 0, \qquad m > 0 \tag{1}$$

Where: P- agricultural monopoly price; Q – agricultural monopoly output; n, m – coefficients of the inverse demand function. Agricultural government-created monopoly will invest in a new information and communication technology whenever doing so lowers its costs. With investement in a new information and communication technology, the firm's marginal cost curve falls to:

$$MC_t = a\,(1\text{-}d) + b\,(1\text{-}d)\,Q_t + c\,(1\text{-}d)Q_t^2 \qquad a< 0\ ,\ b < 0\ ,\ 0 < c < 1,\ 0 < d <1 \tag{2}$$

Where: MC – marginal cost; Q – agricultural monopoly output ; a, b, c – coefficients of the quadratic marginal-cost function, d- the coefficent which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

But because the agricultural monopoly firm faces a downward-sloping demand curve , producing and selling this extra unit also results in a small drop in price (Δ P /Δ Q), which reduces the revenue from all units sold (i.e., a change in revenue Q [Δ P /Δ Q]). Then,

$$MR_t = \frac{\Delta R}{\Delta Q} = \frac{\Delta (P\,Q)}{\Delta Q} = P_t + Q_t\frac{\Delta P}{\Delta Q} = P_t + P_t\left(\frac{Q_t}{P_t}\right)\left(\frac{\Delta P}{\Delta Q}\right) \tag{3}$$

Where: MR – marginal revenue; R – total revenue, P – agricultural monopoly price; Q – agricultural output .

The elasticity of demand is defined as e= (P/Q) (Δ Q / ΔP). The elasticity of demand (e) is negative. Then,

$$MR_t = P_t + P_t\left(\frac{1}{e}\right) = P_t\left[1+\left(\frac{1}{e}\right)\right] \tag{4}$$

Where: MR – marginal revenue; P – agricultural monopoly price; e – the coefficient of the price elasticity of demand. Agricultural monopoly maximizes profit by producing the quantity at which marginal revenue equals marginal cost ($MR_t = MC_t$). Further, it is supposed that:

$$MR_{t+1} = MR_t + \Delta\,MR = MR_t + \alpha\,MR_{t+1} \quad 0 <\alpha< 1 \tag{5}$$

where α – the coefficient of the marginal revenue growthFirstly, it is supposed that a = 0 and n = 0 .
By substitution one derives:

$$P_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}P_t - \frac{c\,(d-1)}{m^2\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}P_t^2 \tag{6}$$

Further, it is assumed that the agricultural monopoly price is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the agricultrual monopoly price growth rate depends on the current size of the agricultural monopoly price, P, relative to its maximal size in its time series P^m. We introduce p as $p = P / P^m$. Thus p range between 0 and 1. Again we index p by t, i.e., write p_t to refer to the size at time steps t = 0,1,2,3,... Now , growth rate of the agricultural monopoly price is measured as

$$p_{t+1} = \frac{b\left(d-1\right)}{m\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t - \frac{c\left(d-1\right)}{m^2\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t^{\,2} \tag{7}$$

This model given by equation (7) is called the logistic model. For most choices of b, c, d, m, α, and e there is no explicit solution for (7). Namely, knowing b, c, d, m, α, and e and measuring p_0 would not suffice to predict p_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz [14] discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

3. The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) , \pi \in [0,4] \quad , \quad z_t \in [0,1] \tag{8}$$

is equivalent to the iteration of growth model (7) when we use the following identification:

$$z_t = \frac{c}{b\,m} p_t \quad \text{and} \quad \pi = \frac{b\left(d-1\right)}{m\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} \tag{9}$$

Using (7) and (9) we obtain

$$z_{t+1} = \left(\frac{c}{b\,m}\right) p_{t+1} = \left(\frac{c}{b\,m}\right)\left\{\frac{b\left(d-1\right)}{m\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t - \frac{c\left(d-1\right)}{m^2\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t^{\,2}\right\} =$$

$$= \frac{c\left(d-1\right)}{m^2\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t - \frac{c^2\left(d-1\right)}{bm^3\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]} p_t^{\,2}$$

On the other hand, using (8), and (9) we obtain

$$z_{t+1} = \pi\, z_t\left(1-z_t\right) = \left\{\frac{b\left(d-1\right)}{m\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]}\right\}\left(\frac{c}{b\,m}\right) p_t \left\{1-\left(\frac{c}{b\,m}\right) p_t\right\}$$

$$= \frac{c\left(d-1\right)}{m^2\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]}p_t - \frac{c^2\left(d-1\right)}{bm^3\left(1-\alpha\right)\left[1+\left(\frac{1}{e}\right)\right]}p_t^{\ 2}$$

Thus we have that iterating (7) is really the same as iterating (8) using (9) .It is important because the dynamic properties of the logistic equation (8) have been widely analyzed (Li and Yorke [13] , May [16]).It is obtained that :(i) For parameter values 0 <π< 1 all solutions will converge to z = 0;(ii) For 1 <π< 3,57 there exist fixed points the number of which depends on π;(iii) For 1 <π< 2 all solutions monotnically increase to z = (π - 1) / π;(iv) For 2 <π< 3 fluctuations will converge to z = (π - 1) / π;(v) For 3 <π< 4 all solutions will continously fluctuate;(vi) For 3,57 <π< 4 the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period.

4. Conclusion

Patents have helped agricultural firms develop new technologies. However, patents create a tradeoff between creating incentives for research and development in agriculture and the inefficiency of the agricultural monopoly.A key hypothesis of this work is based on the idea that the coefficient π (9) plays a crucial role in explaining local stability of the agricultural monopoly price where: b – the coefficient of the marginal cost function of the agricultural monopoly firm, m - the coefficient of the inverse demand function, e - the coefficient of the price elasticity of agricultural monopoly's demand, d - the coefficent which explain effect of the investment in new information and communication technology on the agricultural monopolist's marginal cost curve, α – the coefficient of the marginal revenue growth.

References

[1] Benhabib, J., Day, R.H. Rational Choice and Erratic Behaviour. Review of Economic Studies 1981;48:459-471.
[2] Benhabib, J., Day, R.H. Characterization of Erratic Dynamics in the Overlapping Generation Model. Journal of Economic Dynamics and Control 1982; 4:37-55.
[3] Benhabib, J., Nishimura, K.Competitive Equilibrium Cycles.Journal of Economic Theory 1985;35:284-306.
[4] Day, R.H. Irregular Growth Cycles.American Economic Review 1982;72:406-414.
[5] Day, R.H. The Emergence of Chaos from Classical Economic Growth.Quarterly Journal of Economics 1983;98:200-213.
[6] Day, R.H. Complex Economic Dynamics Volume I: An introduction to dynamical systems and market mechanism. Discrete Dynamics in Nature and Society 1997;1:177-178.
[7] Goodwin, R.M. Chaotic Economic Dynamics. Oxford: Clarendon Press; 1990.
[8] Grandmont, J.M. On Enodgenous Competitive Business Cycles.Econometrica 1985;53:994-1045.
[9] Jablanović D. V. Agricultural Monopolistic Competitior and the Pigovian Tax .Studies in Agricultural Economics 2013;115:57-60. http://dx.doi.org/10.7896/j.1224
[10] Jablanović D. V. The Lerner Index and the Chaotic Monopoly Output Growth Model. Asian Journal of Business and Management Sciences 2012;1(12):102-106.
[11] Jablanović D. V. The Chaotic Long-Run Monopolistic Competitor’s Output Growth.African Journal of Business Management 2012;6(48):11758-11761.
[12] Jablanović D. V. The Chaotic Production Growth Model of the Monopoly Firm .Hyperion International Journal of Econophysics & New Economy 2011;4(2):285-292.
[13] Li, T., Yorke, J. Period Three Implies Chaos. American Mathematical Monthly 1975;8:985-992.
[14] Lorenz, E.N. Deterministic nonperiodic flow. Journal of Atmospheric Sciences 1963;20:130-141.
[15] Lorenz, H.W. Nonlinear Dynamical Economics and Chaotic Motion. 2nd ed. Heidelberg :Springer-Verlag; 1993.
[16] May, R.M. Mathematical Models with Very Complicated Dynamics. Nature 1976;261:459-467.
[17] Medio, A. Chaotic Dynamics: Theory and Applications to Economics. Cambridge: Cambridge University Press; 1993.