Journal of Agricultural Sciences
Vol. 48, No 2, 2003
Pages 159-170

UDC: 631.62:631.445.24
Original scientific paper

APPLICATION OF KRAIJENHOFF VAN DE LEUR – MAASLAND'S METHOD IN DRAINAGE

Nevenka Djurović[1] and Ružica Stričević [1]

Abstract: The aim of this work is to show some properties of the application of Kraijenhoff Van de Leur – Maasland's method for drain spacing determination in unsteady state of flow. The analysis of the method is based on data obtained from drainage field with 10 m of drain spacing which dries out eugley soil. The results of analysis show the range of method applicability as well as certain limitations in the case of non-modelled dynamics of ground water recharges.

Key words: drainage, drain spacing, unsteady state of flow, drain discharge.

Introduction

Kraijenhoff Van de Leur – Maasland's method is used for drain spacing determination in unsteady state of flow. Like other methods used for that purpose, this one also assumes that water head is variable in time, therefore drain recharge is variable in time as well. Water table depth oscillates due to unsteady recharge, therefore variable drain recharge and velocity of water flow in every point of cross section occur as well as variable drain discharge. In fact, this method is not in usage much for practical purpose of drain spacing determination. However, it is very useful for the analysis of water table depth oscillation and variation of drain discharge velocity as a consequence of recharge. Besides its advantages, this method in unsteady state of flow in different circumstances has some peculiarities, which could be considered as a limitation in its application. The aim of this work is to find out advantages and limitations of the Kraijenhoff van de Leur – Maasland's method in unsteady state of flow in eugley type of soil.

[1] Nevenka Djurović, PhD., Assistant Professor, Ružica Stričević, PhD., Professor, Faculty of Agriculture, 11081 Belgrade-Zemun, Nemanjina 6, Serbia and Montenegro

Material and Methods

Experimental filed is situated in Radmilovac on eugley type of soil, near Belgrade . Water logging occurred every year from autumn to mid-spring on the area of 1.5 hectare. Average value of hydraulics conductance is 0.6 m·day To dry out the soil, subsurface pipe drainage system was installed. Drain spacing of 10 m represented drainage treatments. Average drain depth is 0.9 m..

Kraijenhoff van de Leur (1958) and Maasland (1959) derived new solution for unsteady state of flow to the drain (ILRI I – IV). Starting from the flat water level in drains in time t = 0 and assuming recharge intensity R (m·day^{-1}) from the moment t = 0, they established the starting boundary conditions as follows:

$h = 0$ for $t = 0$ if $0 < x < L$ - initial horizontal groundwater table at drain level at $t = 0$
$h = 0$ $t > 0$ if $x = 0$; $x = L$ -water table depth in drain is unchanged (in the level of drain), water recharge in drain is steady from the beginning ($t = 0$)
R = const. $t > 0$ - steady water recharge which starts in $t = 0$

For the above mentioned assumed initial and boundary conditions, water table depth in the middle distance from two drains ($x = L/2$) at any time (t) can be expressed as

$$h_t = \frac{4}{\pi}\frac{R}{\mu} j \sum_{n=1,-3,5}^{\infty} \frac{1}{n^3}\left(1 - e^{-n^2 \frac{t}{j}}\right) \tag{1}$$

where: h_t – height of water depression curve (m)
μ - drainage porosity (·)
k – hydraulic conductivity of layer (mm·day^{-1})
t – time of drainage (days)

where

$$j = \frac{\mu L^2}{\pi^2 KD} \tag{2}$$

The expression j is called coefficient of reservoir, since it represents total capacity of aquifer after infinite long time, hence at steady state of flow. Factor j is:

$$j = \frac{1}{\alpha}$$

where α - is a factor of reaction by Dumm and Zeeuw.

Intensity of drain discharge q_t (mm·day^{-1}) of a parallel subsurface pipe drainage system at any time can be obtained from the expression:

$$q_t = \frac{8}{\pi^2} R \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \left(1 - e^{-n^2 \frac{t}{j}} \right) \tag{3}$$

Equations 2 and 3 are valid in steady state of recharge R. If such recharge lasts long enough, and water flow to the drain continues to be steady for $t \to \infty$, equations 3 becomes

$$q = \frac{8}{\pi^2} R \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} = \frac{8}{\pi^2} R \frac{\pi^2}{8} = R \tag{4}$$

which referes to steady state conditions, therefore intensity of recharge is equal to intensity of drain discharge.

For $t \to \infty$ equation becomes:

$$h = \frac{4}{\pi} \frac{R}{\mu} j \sum_{n=1,-3,5}^{\infty} \frac{1}{n^3} = \frac{4}{\pi} \frac{R}{\mu} j \frac{\pi^3}{32} = \frac{\pi^2}{8} \frac{R}{\mu} j \tag{5}$$

Substitution of j from the equation gives:

$$h = \frac{RL^2}{8KD} \tag{6}$$

Equivalent depth introduction will change the equation into:

$$j = \frac{1}{\alpha} = \frac{\mu L^2}{\pi^2 Kd} \tag{7}$$

Application of Kraijenhoff van de Leur – Maasland's method

This equation is more applicable for water table depth oscillation analysis and variation of drain discharge velocity as a consequence of recharge than for drain spacing determination. Nevertheless, equation can be applied in three cases:

1. constant and continual recharge
2. constant recharge followed by period of restriction
3. intermittent recharge

Case 1. Constant and continual recharge

Equations 1 and 3 can be expressed as follows:

$$h_t = \frac{R}{\mu} j c_t \tag{8}$$

where

$$c_t = \frac{4}{\pi} \sum_{n=1,-3,5}^{\infty} \frac{1}{n^3}\left(1 - e^{-n^2 \frac{t}{j}}\right) \tag{9}$$

and

$$q_t = R g_t \tag{10}$$

where

$$g_t = \frac{8}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\left(1 - e^{n^2 \frac{t}{j}}\right) \tag{11}$$

Parameters c_t and g_t depend only on time *t,* reservoir coefficients *j* or on the factor of reaction α. Determination of c_t and g_t is based on the table as given by the authors of the method.

Case 2. Constant recharge followed by period of restriction

The application of equation under this conditions could be presented as a case of discharge area under irrigation or rainfall occurring during one single day after drought period. During that day at any time $t < 1$ day both drain discharge and height of water table could be calculated from the above solution. However, for $t > 1$ day solution is not valid because the condition of a constant recharge is not fulfilled. In order to be able to use the solutions for $t > 1$ day, we assume that the recharge of the first day continues throughout the following days, but from the second day onwards (fig. 1) an equal negative recharge –R occurs, so that the total recharge for $t > 1$ day is equal to zero (principle of superposition).

For the water table depth at the end of the first day (t = 1), the expression is:

$$h = h_1 = \frac{R}{\mu} j c_1 \tag{12}$$

At the end of the second day positive recharge over two days occurred, therefore:

$$h_2^{`} = \frac{R}{\mu} j c_2 \tag{13}$$

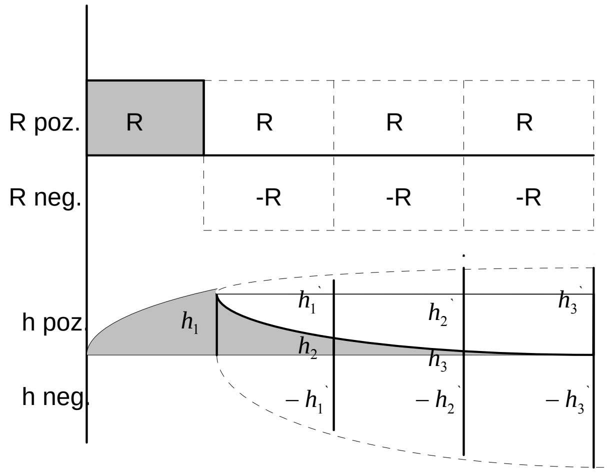

Fig. 1. - Principle of superposition recharge and elevation of water table depth for the Kraijenhoff an de Leur – Maasland's equation

From the previous expression negative effect of the first day discharge should be extracted as:

$$h_1^{`} = h_1 = \frac{R}{\mu} jc_1 \tag{14}$$

to obtain equation:

$$h_2 = h_2^{`} - h_1^{`} = \frac{R}{\mu} j(c_2 - c_1 \tag{15}$$

Similar effect occurred at the end of the third day:

$$h_3^{`} = \frac{R}{\mu} jc_3 \tag{16}$$

$$h_2^{`} = \frac{R}{\mu} jc_2 \tag{17}$$

so that:

$$h_3 = h_3^{`} - h_2^{`} = \frac{R}{\mu} j(c_3 - c_2) \tag{18}$$

Finally, at the end of t^{-1} day the equation will be:

$$h_t = h_t^{`} - h_{t-1}^{`} = \frac{R}{\mu} j(c_t - c_{t-1}) \tag{19}$$

In this case, the water table depth can be calculated using the table given by the authors of the method (Wesseling, 1977).

Case 3. Intermittent Recharge

The above mentioned case can be applied in a more general way if intermittent recharge occurred. The application of the method is described by the sample of recharge shown in figure 2. Suppose that water table depth calculation is needed or discharge at the end of *m* day if previous days variable recharge is recorded. Water table depth as well as drain discharge are influenced by water percolation every previous day. Therefore, recharges of every day from m-3 till m should be taken into consideration.

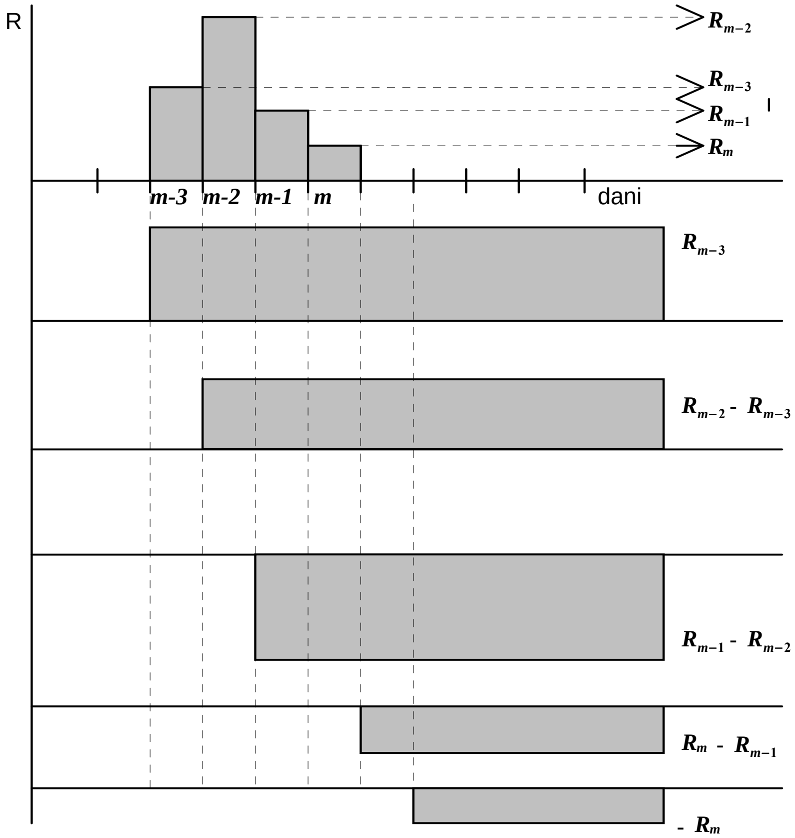

Fig.2.- Application of Kraijenhoff van de Leur-Maasland's method for the variable drain discharge in time

Height of groundwater table is:

$$h_m = \frac{j}{\mu}\left[R_m c_1 + R_{m-1}(c_2 - c_1) + R_{m-2}(c_3 - c_2) + \ldots R_1(c_m - c_{m-1})\right] \qquad (20)$$

Supposing that $C_1 = c_1 j$, $C_2 = (c_2 - c_1) j$, $\dots C_m = (c_m - c_{m-1}) j$, the following expression can be obtained:

$$h_m = \frac{1}{\mu}\left[C_1 R_m + C_2 R_{m-1} + C_3 R_{m-2} + \dots C_m R_1\right] \tag{21}$$

Similar principle can be applied to discharge as well:

$$q_m = G_1 R_m + G_2 R_{m-1} + G_3 R_{m-2} + \dots G_m R_1 \tag{22}$$

where $G_1 = g_1$, $G_2 = (g_2 - g_1)$, $G_m = (g_m - g_{m-1})$

Factors $C_1, C_2, \dots G_1, G_2$ are the functions $\alpha=1/j$ given in tables (Wesseling, 1977).

To apply this method for drain spacing determination according to the measured data of water table depth or drain discharge, numerical method is needed, because drain spacing cannot be expressed as an explicit function of measurement. If equation 20 is expressed in the forms:

$$h_m = \frac{j}{\mu} f_1\left(R_1, R_2, \dots, R_m, c_1, c_2, \dots, c_m\right) \tag{23}$$

where:

$$j = \frac{\mu L^2}{\pi^2 K d}, \tag{24}$$

$f_1(\cdot)$ is the function of $2m$ variable and each of the items c_i ; $i = 1, \dots, m$ is a function of parameter j, with reference to L, the following relation can be obtained:

$$h_m = C L^2 f_2(L) \tag{25}$$

where C is some constant value and $f_2(\cdot)$ complex function , from which value of drain spacing cannot be explicitly expressed. To solve that problem, relation 25 can be expressed in the form:

$$C L^2 f_2(L) - h_m = f_3(L) = 0 \tag{26}$$

Suppose that the function $f_3(\cdot)$ are two time differentiable on the segment $[a, b]$, where square root of L^* equation is on. To equation $f_3(L)$ equivalent equation can be added,

$$L = L - \frac{f_3(L)}{f_3^{`}(L)} \tag{27}$$

by which iterative procedure can be formed:

$$L_{k+1} = L_k - \frac{f_3(L_k)}{f_3^{`}(L_k)} \tag{28}$$

The above principle of solving non-linear function is well known as Newton-Raphson's method and it shows quadratic convergence. This method is used to estimate drain spacing.

Results and Discussion

The application of Kraijenhoff van de Leur-Maasland's method in drainage of marshy gley soil.

For the analysis of Kraijenhoff van de Leur-Maasland's method for drain spacing estimation, sequences of measurement in the range of unsteady state of flow were taken in this work.

T a b. 1.- Sequence of groundwater height measurement above the axis of the drain and drain discharge upon which is carried out drain spacing estimation by Kraijenhoff van de Leur-Maasland's method

Index measurement	Data	*h* (m)	*q* (m/dan)	Index measurement	Data	*h* (m)	*q* (m/dan)
1	21 Nov.96	0.02	0.00081	21	25 Jan.97	0.52	0.00617
2	24 Nov.96	0.35	0.0092	22	29 Jan.97	0.48	0.00906
3	27 Nov.96	0.41	0.0198	23	2 Feb.97	0.39	0.01326
4	30 Nov.96	0.55	0.02406	24	6 Feb.97	0.4	0.01423
5	2 Dec.96	0.55	0.0123	25	10 Feb.97	0.35	0.0108
6	5 Dec.96	0.7	0.03328	26	14 Feb.97	0.36	0.00921
7	8 Dec.96	0.67	0.01114	27	16 Feb.97	0.38	0.0124
8	11 Dec.96	0.56	0.0145	28	19 Feb.97	0.48	0.00773
9	15 Dec.96	0.57	0.0112	29	22 Feb.97	0.52	0.00699
10	19 Dec.96	0.51	0.0159	30	26 Feb.97	0.41	0.00844
11	22 Dec.96	0.48	0.00904	31	2 Mar.97	0.15	0.00413
12	25.Dec.96	0.52	0.01752	32	8 Mar.97	0.14	0.00407
13	29 Dec.96	0.56	0.01112	33	11 Mar.97	0.1	0.00161
14	3 Jan.97	0.54	0.0109	34	14 Mar.97	0.05	0.00099
15	6 Jan.97	0.54	0.01712	35	17 Mar.97	0.06	0.00099
16	10 Jan.97	0.56	0.01254	36	20 Mar.97	0.11	0.00161
17	12 Jan.97	0.56	0.00965	37	22 Mar.97	0.1	0.00162
18	14 Jan.97	0.57	0.00994	38	26 Mar.97	0.1	0.0059

19	17 Jan.97	0.56	0.00714	39	30 Mar.97	0.1	0.0051
20	21 Jan.97	0.56	0.00715				

Annotation**:** *h*- groundwater heights above the axis of the drain**,** *q* -drain discharge

To analyze this method, conditions of unsteady state of flow should be defined precisely. Especially, series of estimation should start at the moment when groundwater height is in the level of drain, therefore $h = 0$. Besides this, it is not necessary that groundwater table depletion occurs and absence of rainfall is not needed either, because the principle of superposition applied in the method takes all that into consideration. The measured data which fulfilled the above mentioned conditions are shown in Table 1. It will be used in the analysis of Kraijenhoff van de Leur-Maasland's method. Responsive data of drain discharge are added, too.

In figures 3 and 4 are shown the results of drain spacing estimation by Kraijenhoff van de Leur-Maasland's method. Drain spacing estimation shown in Figure 3 is much larger than it is in reality. This can be explained by the assumption of Kraijenhoff van de Leur-Maasland's method which considers that the only recharge occurred under rainfall or irrigation. Taking into account that the type of soil of the experimental field abounds in ground waters, therefore there were additional, but not measured recharges. In this case, the method explained (expressed) by fictive, larger drain spacing depletion of ground water. The importance of possessing exact measurements of all recharge sources made this method less applicable in practice. In other words, there are not many cases where all the requirements are fulfilled to apply this method.

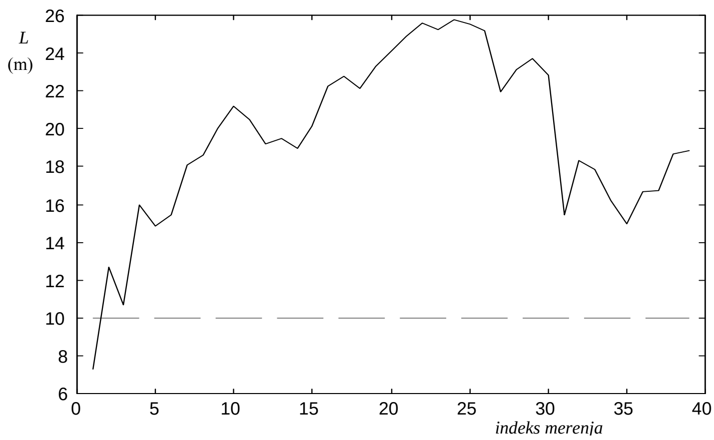

Fig. 3.- Drain spacing estimation by Kraijenhoff van de Leur- Maasland's method (L=10 m)

Non-suitability of this method, especially for the narrow drain spacing (10 m) is confirmed by the high value of error estimation (9.62 m), high value of median

(9.503 m) and value of mod of error (8.363 m). The shape of the histogram, shown in Figure 4, refers to the systematic nature of error: significant bias from the normal distribution of error (non-smooth function when argument is biased in regard to the estimated mean value). Histogram of error shows a high error, considering that the most concentration of data are in the realm of around 8m.

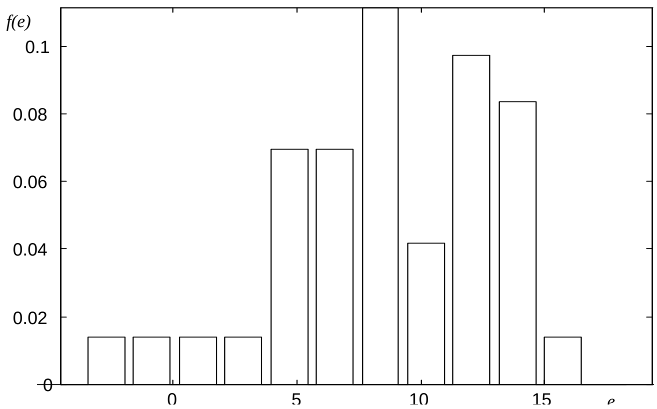

Fig. 4.- Histogram of error of estimation by applying Kraijenhoff van de Leur- Maasland's method (L = 10 m)
Note: *f*(e)-probability density function of *e*; *e*-error of estimation

By non-feasibility of any source of recharge measurements, the method estimates larger drain spacing than it is in the field. Similar results were obtained by applying other methods for drain spacing determination in unsteady state of flow in eugley type of soil, such as Glover -Dumm (Djurović et al. 2000). Non-registered sources of recharge and their magnitude are less emphasised in the case of drainage system with larger drain spacing. Hence, applicability of the method is better in larger drain spacing estimation. In other words, in the system with larger drain spacing, the effect of non modelled dynamic of recharge has less influence, therefore the method can be successfully applied under certain limitations.

Conclusion

The method of Kraijenhoff van de Leur-Maasland can be used for the analysis of ground water level oscilations and variations in velocity of drain discharge as a consequence of recharge variation in unsteady state of flow. It considers that the only sources of recharge are rainfall or irrigation. Considering that marshy gley soil abounds in ground water, there are recharges regardless on rainfalls or irrigation. As compared with Glover-Dumm method , this one explained (expressed) ground water depletion by fictive, larger drain spacing methods. The importance of possessing exact measurements of all recharge sources made this method less applicable in practice. It is more or less common for almost all methods applicable in unsteady state of flow.

REFERENCES

1. ILRI I (1979): Drainage principles and Application Vol. I: Introductory subjects Wageningen, The Netherlands.
2. ILRI II (1980): Drainage principles and Application Vol. II: Theories of field drainage and watershed runoff, Wageningen, The Netherlands.
3. ILRI III (1980): Drainage principles and Application Vol. III: Surveys and investigation. Wageningen, The Netherlands.
4. ILRI IV (1980): Drainage principles and Application Vol. IV: Design and management of drainage systems. Wageningen, The Netherlands.
5. Djurović, N., Stričević, R., Gajić, B., (2000): Some constraints of the application of methods for drain spacing determination in unsteady- state of flow in eugley soil. Review of Research Work at the Faculty of Agriculture, Vol 45, No 2, pp 83-91.
6. Wesselng, J. (1974): Drainage Principles and Practices, Vol. II ILRI Wageningen. The Netherlands.

Received July 11, 2003
Accepted November 4, 2003

PRIMENA METODE KRAIJENHOFF VAN DE LEUR-MAASLAND U ODVODNJAVANJU

Nevenka Djurović[1] i Ružica Stričević [1]

Rezime

Medju metodama za odredjivanje medjudrenskog rastojanja u nestacionarnom režimu filtracije je i metoda Kraijenhoff van de Leur-Maasland. Kao i druge metode iz ove grupe, ona polazi od predpostavke da je priticanje u dren promenljivo u toku vremena, pa se i pritisak pod kojim se isticanje dešava menja u toku vremena. Nivo podzemnih voda osciluje u vremenu, i to pod uticajem nestalnog priticanja, uglavnom od padavina. Metoda Kraijenhoff van de Leur-Maasland se zapravo ne koristi za praktična izračunavanje rastojanja izmedju

[1] Dr Nevenka Đurović, docent, dr Ružica Stričević, vanredni profesor, Poljoprivredni fakultet , 11081 Beograd-Zemun, Nemanjina 6, Srbija i Crna Gora

drenova. Medjutim, korisna je kod analize promene nivoa podzemnih voda i promena brzine isticanja koje su posledica promena doticaja. Jednačina se može primenjivati u tri slučaja: pri konstantnom i kontinualnom doticanju, pri konstantnom doticanju praćenom periodom restrikcije, i u slučaju kada je doticanje proizvoljna funkcija vremena. Procene medjudrenažnog rastojanja na drenažnom sistemu sa L=10m su veće nego što ono zaista jeste. Metoda van de Leur-Maasland-a podrazumeva da doticaja, osim merenog, dakle padavina ili navodnjavanja, nema. S obzirom na to da ovo zemljište obiluje podzemnim vodama, dakle postoje doticaji koji nisu obuhvaćeni niti kroz padavine niti kroz navodnjavanje, ova metoda pokušava da smanjene dubine podzemnih voda obrazloži fiktivnim, većim rastojanjem izmedju drenova. Na sistemima sa većim medjudrenažnim rastojanjima, efekat nemodelirane dinamike doticaja ima manje uticaja. Ova osobina metode van de Leur-Maasland-a pokazaće se kao veoma veliko ograničenje u primeni, jer zahteva egzaktno merenje i poznavanje svih doticaja vode. U ovom smislu metoda pokazuje sličnost sa drugim metodama za odredjivanje rastojanja izmedju drenova u nestacionarnom režimu filtracije.

Primljeno 11. jula 2003.
Odobreno 4. novembra 2003.